Impossibility of Measuring the Wave Function of a Single Quantum System

G. M. D'Ariano* and H. P. Yuen

Department of Electrical Engineering and Computer Science, Department of Physics and Astronomy, Northwestern University, Evanston, Illinois 60208

(Received 6 September 1995)

The general impossibility of determining the state of a single quantum system is proved for arbitrary measuring schemes, including a succession of measurements. Some recently proposed methods are critically examined. A scheme for tomographic measurements on a single copy of a radiation field is devised, showing that the system state is perturbed however weak the system-apparatus interaction is, due to the need of preparing the apparatus in a highly "squeezed" state.

PACS numbers: 03.65.Bz, 42.50.Dv, 42.65.Ky

Recently, the possibility of determining the wave function of a single quantum system has been debated by several authors [1–5], exploring concrete measurement schemes based on vanishingly weak quantum nondemolition measurements [1], weak measurements on "protected" states [2], "logically reversible" [3], and "physically reversible" [4,5] measurements. In each of these schemes the conclusion is that it is practically impossible to measure the wave function of a single system, either because the weakness of the measuring interaction prevents one from gaining information on the wave function [1] or because the method of protecting the state [2] actually requires some *a priori* knowledge on the state (this is suggested in Refs. [5] and [1]), or because quantum measurements can be physically reverted only with a probability of success equal to $1/2$ [5].

In this Letter, we will show the impossibility of any measurement scheme for determining the wave function from a single copy of the system. Despite its fundamental relevance in the logical framework of quantum mechanics, it seems that the impossibility of measuring a single-system wave function has never been proved in general. On the basis of a simple argument—the "cloning machine"—we will prove that such a possibility would contradict the most basic assumption of quantum mechanics, namely, unitarity. Moreover, we demonstrate that any sequence of measurements on the same system cannot yield more information on its state than just an appropriately chosen single measurement, and we show explicitly how both the "state protection" and the "reversible measurements" methods fall under such general consideration. These two first parts together cover all possible transformations in quantum physics. In order to gain physical insight on why vanishingly weak repeated measurements cannot be successful, we consider the case of "quantum homodyne tomography" [6,7], which recently has been shown to be a genuine measurement of the density matrix of the radiation field [8]; in this case the field is prepared in the same state at each measurement, in agreement with the usual statistical meaning of the quantum state based on an ensemble of identical systems. From Ref. [9] we learned that a minimum nonvanishing quantum efficiency of the detector $\eta_* = \frac{1}{2}$ is needed for measuring the density matrix. Here, we show that the existence of a lower bound η_* is of fundamental relevance, because it prevents one from measuring the state of a single system. In fact, when a repeatable homodyne scheme is devised, its effective quantum efficiency vanishes for a weak system-apparatus interaction. Thus, in order to overcome low quantum efficiency, the apparatus must be prepared in a highly squeezed state which, by itself, amplifies the backaction to a finite extent. Hence, however weak the system-apparatus interaction is, on average the system is always perturbed by a finite amount, or else no information is gained.

The "cloning machine" argument.—A quantum cloning machine [10,11] is a device that is capable of producing $n > 1$ copies of a generic state $|\psi\rangle$ from a given set of possible states which may be the whole space. It has to be represented by a state transformation of the general form

$$|v\rangle \otimes |\psi\rangle \otimes |\omega_1\rangle \otimes \cdots \otimes |\omega_{n-1}\rangle \rightarrow |v'(\psi)\rangle \otimes \underbrace{|\psi\rangle \otimes \cdots |\psi\rangle}_{n \text{ times}}, \quad (1)$$

where both the transformation and the state preparation must be independent of $|\psi\rangle$—which *a priori* is unknown. In Eq. (1) $|\omega_1\rangle \otimes \cdots \otimes |\omega_{n-1}\rangle$ denotes the state preparation of the modes that support clones, whereas $|v\rangle$ is the initial state of sufficiently many other modes—including the apparatus and the environment—so that the transformation (1) can be taken unitary. All states are supposedly normalized. For simplicity, a pure state for the environment has been used: the argument can be easily extended to a mixed state. We now show that if the system is known *a priori* to be in any one of two known nonorthogonal states, then the possibility of exactly determining merely which one of the two states already violates unitarity. Consider the cloning of two nonorthogonal states—say $|\psi\rangle$ and $|\varphi\rangle$, with $0 < |\langle\psi|\varphi\rangle| < 1$: the transformation (1) must preserve the scalar product in order to be unitary. Taking the scalar product of the two

 0031-9007/96/76(16)/2832(4)$10.00

states on the left-hand side of Eq. (1) for $|\psi\rangle$ and $|\varphi\rangle$, and equating the result to the scalar product of the two corresponding states on the right, we obtain the contradictory identity $\langle v'(\psi)|v'(\varphi)\rangle\langle\psi|\varphi\rangle^{n-1} = 1$, which would require $|\langle v'(\psi)|v'(\varphi)\rangle| > 1$ for $n > 1$. Hence, *reductio ab absurdum,* the transformation (1) cannot be unitary. (The same conclusion follows for antiunitary transformations.) Thus one cannot clone the state of a system known to be in *any one* of two nonorthogonal states, say, $|\psi\rangle$ and $|\varphi\rangle$, using a single state-independent unitary transformation. Let us now suppose that there is a device that is capable of determining (by a unitary process) in which of these two states the system is. Then, once the state is known, one can easily choose unitary transformations that generate an n clone of $|\psi\rangle$ or $|\varphi\rangle$, depending on the two possible results of this device. Hence, such a "state detector" would lead to a realization of the cloning machine. In this way it is proven that the possibility of distinguishing two nonorthogonal states contradicts the unitarity of quantum mechanical transformations.

Repeated versus single measurements.—Using an apparatus that interacts very weakly with the system, one could devise measurement schemes involving repeated measurements, which, on first sight, seem to allow one to retrieve more information on the quantum state of a single system than that obtained by a single measurement. In fact, they turn out not to, often for rather subtle reasons. Examples were given in [1,2,5], and we shall later give another one. Here we will show that no succession of repeated measurements performed on a single system can retrieve more information than an appropriately chosen single measurement whose output state is independent of the input one. Incidentally, this also shows that the single measurement formulation in quantum detection theory [12] entails no loss of generality.

In order to have an output state that depends on the state before the measurement, the measurement scheme must involve a probe that interacts with the system and later is "measured" to yield information on the original state of the system [13]. This *indirect* measurement scheme is completely specified once the following ingredients are given: (i) the unitary operator $\hat{U}$ that describes the system-probe interaction, (ii) the state $|\varphi\rangle$ of the probe before the interaction, and (iii) the observable $\hat{X}$ which is measured on the probe. At the end of the system-probe interaction it is possible to consider another measurement on the system, say the measurement of an observable $\hat{Y}$ (for the sake of notation we take $\hat{X}$ and $\hat{Y}$ both having continuous spectrum, with eigenvectors $|x\rangle$ and $|y\rangle$, respectively). The conditional probability density $p(y|x)$ of getting a result y from the second measurement given the result of the first one being x can again be written in terms of the Born's rule $p(y|x)dy = \langle y|\hat{\varrho}_x|y\rangle$ upon defining a "reduced state" $\hat{\varrho}_x$ as follows:

$$\hat{\varrho}_x = \frac{\hat{\Omega}(x)\hat{\varrho}\hat{\Omega}^\dagger(x)}{\mathrm{Tr}[\hat{\varrho}\hat{\Omega}^\dagger(x)\hat{\Omega}(x)]}, \tag{2}$$

where the system operator $\hat{\Omega}(x)$ is given by

$$\hat{\Omega}(x) = \langle x|\hat{U}|\varphi\rangle, \tag{3}$$

and

$$d\hat{\mu}(x) = \hat{\Omega}^\dagger(x)\hat{\Omega}(x)\,dx \tag{4}$$

is a "probability operator-valued measure" (POM) [13]. It provides the Born's rule for the measurement in the form

$$p(x)\,dx = \mathrm{Tr}[\hat{\varrho}\,d\hat{\mu}(x)]. \tag{5}$$

Equation (5) is also the most general form of the Born's rule for any kind of measurement, with $d\hat{\mu}(x)$ a POM not necessarily given in the form (4). In the present situation, the most general representation of a quantum measurement—the so-called "instrument" map [13]—reduces to

$$\hat{\varrho} \to dI(x)\hat{\varrho} \doteq \hat{\Omega}(x)\hat{\varrho}\hat{\Omega}^\dagger(x)\,dx, \tag{6}$$

which provides both Born's rule $p(x)\,dx = \mathrm{Tr}[dI(x)\hat{\varrho}]$ and the state reduction $\hat{\varrho} \to \hat{\varrho}_x = dI(x)\hat{\varrho}/\mathrm{Tr}[dI(x)\hat{\varrho}]$. A sequence of N indirect measurements is described by the composition of their respective instruments, corresponding to the operator $\hat{\Omega}^{(N)}(\mathbf{x}) = \hat{\Omega}_N(x_N)\cdots\hat{\Omega}_2(x_2) \times \hat{\Omega}_1(x_1)$, with $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ and the setup $\{\hat{U}_n, |\varphi_n\rangle, \hat{X}_n\}$ generally different at every measuring step. Now we show that the same POM $d\hat{\mu}(\mathbf{x}) = [\hat{\Omega}^{(N)}(\mathbf{x})]^\dagger\hat{\Omega}^{(N)}(\mathbf{x})d^n\mathbf{x}$ can be achieved by a single measurement whose output state is independent of the input one. Indeed, according to Naimark's theorem [12] any POM $d\hat{\mu}(\mathbf{x})$ on the Hilbert space $\mathcal{H}$ can be achieved by a measurement of self-adjoint commuting operators acting on a suitably extended Hilbert space $\mathcal{H} \otimes \mathcal{H}_P$. One has $d\hat{\mu}(\mathbf{x}) = \mathrm{Tr}_P[\hat{1} \otimes |F\rangle\langle F|\,|\mathbf{x}\rangle\langle\mathbf{x}|]$, with Tr_P denoting the partial trace over the (probe) Hilbert space $\mathcal{H}_P$, $|\mathbf{x}\rangle$ being simultaneous eigenvectors of the commuting observables, and $|F\rangle$ a suitable probe preparation. This POM can be achieved, for example, by the instrument $dI(x)\hat{\varrho} = \mathrm{Tr}_P[|\lambda_\mathbf{x}\rangle\langle\mathbf{x}|\hat{\varrho} \otimes |F\rangle\langle F|\,|\mathbf{x}\rangle\langle\lambda_\mathbf{x}|]$ where $|\lambda_\mathbf{x}\rangle$ is any family of normalized vectors in $\mathcal{H} \otimes \mathcal{H}_P$. For such an instrument the state after the measurement is $\hat{\varrho}_\mathbf{x} = \mathrm{Tr}_P[|\lambda_\mathbf{x}\rangle\langle\lambda_\mathbf{x}|]$, which is independent of the state before the measurement. Thus we conclude that any succession of repeated measurements performed on a single system gives exactly the same probability distribution of an appropriately chosen single measurement with the output state independent of the input one.

Regarding the possibility of "reversible" measurements [3–5] note that the measurement can be reversed with probability one for any (*a priori* unknown) state $\hat{\varrho}$ only if the operator $\hat{\Omega}(x)$ in Eq. (3) is unitary, and this can only be achieved by an interaction Hamiltonian $\hat{H}_I = f(\hat{O}_P, \hat{O}_S, \hat{O}'_S, \ldots)$ which is a function of a single probe operator $\hat{O}_P$ (and of any number of system operators) with either $|x\rangle$ or $|\psi\rangle$ in Eq. (3) an eigenvector of $\hat{O}_P$. In this case, however, the POM (4) is trivially a multiple of the identity, and the probability distribution does not depend on the state and thus is no measurement at all. Regarding the "protective measurements" in Ref. [2], they are achieved by Hamiltonians of the form $\hat{H}_0 + \kappa\hat{P}\hat{O}_S$ in the limit of κ vanishingly small compared to the strength of the "protecting" free Hamiltonian $\hat{H}_0$, and with the probe operator $\hat{P}$ conjugate to the measured operator $\hat{X}$. Then, it is easy to show that for interaction time t one has $\hat{\Omega}(x) = \exp(-i\hat{H}_0 t/\hbar)[\varphi(x) - \kappa\partial_x\varphi(x)\int_0^t d\tau \times \hat{O}_S(\tau)] + \mathcal{O}(\kappa^2)$, with $\varphi(x) = \langle x|\varphi\rangle$: Hence, in the limit $\kappa \to 0$ the operator $\hat{\Omega}(x)$ leaves invariant only the eigenstates of $\hat{H}_0$, which, however, constitute an orthogonal set. Note that the possibility of distinguishing among *orthogonal* states is not ruled out by the "cloning machine" argument: In fact, it is possible to duplicate a state known *a priori* to be *any one* of a known orthogonal set [14].

Quantum tomography on a single system.—Optical homodyne tomography [6] is a method for determining the matrix elements $\langle\psi|\hat{\varrho}|\psi'\rangle$ of the density operator $\hat{\varrho}$ of the electromagnetic field between vectors $|\psi\rangle$ and $|\psi'\rangle$, preparing the field again in the same state at each measurement. For a single mode field with annihilator operator a the matrix elements are obtained by making repeated measurements of the quadrature operators $\hat{a}_\phi = \frac{1}{2}(a^\dagger e^{i\phi} + ae^{-i\phi})$ at different phases ϕ. Before analyzing a repeatable measurement scheme for a single system, let us briefly recall the basics of the method [9].

The density operator is connected to the probabilities $p(x,\phi)$ of the outcomes of the quadratures $\hat{a}_\phi$ according to the identity [9]

$$\hat{\varrho} = \int_0^\pi \frac{d\phi}{\pi}\int_{-\infty}^{+\infty} dx\, p(x,\phi)K_\eta(x - \hat{a}_\phi)\,, \quad (7)$$

where the kernel $K_\eta(x)$ given by

$$K_\eta(x) = \frac{1}{2}\,\mathrm{Re}\int_0^{+\infty} dr\, r\exp\!\left(\frac{1-\eta}{8\eta}\, r^2 + irx\right) \quad (8)$$

depends parametrically on the detector quantum efficiency η. In a real experiment, according to Eq. (7) the density matrix elements $\langle\psi|\hat{\varrho}|\psi'\rangle$ are measured by averaging the kernels $\langle\psi|K_\eta(x - \hat{a}_\phi)|\psi'\rangle$ over the experimental data (x,ϕ), provided that $\langle\psi|K_\eta(x - \hat{a}_\phi)|\psi'\rangle$ are bounded as a function of x and ϕ. As shown in Ref. [9] the matrix elements $\langle\psi|K_\eta(x - \hat{a}_\phi)|\psi'\rangle$ can be bounded although the kernel $K_\eta(x)$ is an unbounded function (not even a tempered distribution). For example, for the number representation ($|\psi\rangle$ and $|\psi'\rangle$ number states) the kernel matrix elements are bounded when $\eta > \frac{1}{2}$. More generally, for any representation there is a lower bound η_* above which the density matrix can be obtained, and, at minimum $\eta_* = \frac{1}{2}$. Hence, in order to measure the density matrix by homodyne tomography, one needs a sizable quantum efficiency $\eta > \eta_*$.

The above tomographic scheme requires preparation of the field in the same state at every measurement, because the radiation is completely absorbed by the detector. Now we devise an indirect scheme that allows repeated measurement on the same physical field. It goes by measuring the quadrature $\hat{a}_\phi$ through the quadrature $\hat{b}_\phi$ of another "probe" mode b interacting with a. Without loss of generality, we consider that before every single measurement the probe is prepared in a pure state $|\upsilon(\phi)\rangle$, which we have the freedom to choose as a function of ϕ. The generating function of the moments of $\hat{b}_\phi$ after the interaction with a is given by

$$X(\lambda,\phi) = \mathrm{Tr}[e^{i\lambda\hat{b}_\phi}\hat{U}\hat{\varrho}\otimes|\upsilon(\phi)\rangle\langle\upsilon(\phi)|\hat{U}^\dagger]\,, \quad (9)$$

and is just the Fourier transform of the probability distribution of the experimental outcomes. We consider the interaction [15] $\hat{U} = \exp[\kappa(ab^\dagger - a^\dagger b)]$, which describes the field transformation at a beam splitter

$$\hat{U}^\dagger b\hat{U} = \sin\kappa a + \cos\kappa b \equiv \varepsilon^{1/2}a + (1-\varepsilon)^{1/2}b\,, \quad (10)$$

where ε is the mirror transmissivity, and b is the annihilation operator of the field mode of the unused port at the same frequency of a. Thus the present scheme corresponds physically to letting the field mode a shine over a chain of many low-transmissivity mirrors and detect the quadrature of the weak transmitted field at each mirror. From the linearity of Eq. (10), the generating function $X(\lambda,\phi)$ factorizes into the product of the generating functions pertaining to modes a or b separately $(\chi_a(\lambda,\phi) = \mathrm{Tr}[\exp(i\lambda\hat{a}_\phi)\hat{\varrho}]$ and similarly $\chi_b(\lambda,\phi))$, namely,

$$X(\lambda,\phi) = \chi_a(\varepsilon^{1/2}\lambda,\phi)\chi_b((1-\varepsilon)^{1/2}\lambda,\phi)\,. \quad (11)$$

Using Eq. (11), by the same methods of Refs. [8,9] it is easy to obtain the identity

$$\hat{\varrho} = \int_0^\pi \frac{d\phi}{\pi}\int_{-\infty}^{+\infty} dx\, p_\varepsilon(x,\phi)\Xi_\varepsilon(x - \hat{a}_\phi)\,, \quad (12)$$

where $p_\varepsilon(x,\phi)$ is the probability of the measured quadrature $\hat{b}_\phi$ rescaled by $\varepsilon^{1/2}$ (as in customary homodyning, where the output photocurrent is rescaled by the quantum efficiency)

$$p_\varepsilon(x,\phi) = \varepsilon^{1/2}\int_{-\infty}^{+\infty}\frac{d\lambda}{2\pi}\, e^{-i\lambda\varepsilon^{1/2}x}X_\theta(\lambda,\phi)\,, \quad (13)$$

and the kernel $\Xi_\varepsilon(x)$ is given by

$$\Xi_\varepsilon(x) = \frac{1}{2}\,\mathrm{Re}\int_0^{+\infty} d\lambda\,\lambda e^{i\lambda x}[\chi_b(\lambda\sqrt{(1-\varepsilon)/\varepsilon}\,,\phi)]^{-1}, \quad (14)$$

and generally depends on the coupling parameter ε and on the probe state $|\upsilon(\phi)\rangle$. One can easily see that when the probe mode b is in the vacuum state the kernel (14) is identical to $K_\eta(x)$ in Eq. (8) with $\varepsilon \equiv \eta$, namely, the transmissivity ε plays the role of the overall quantum efficiency of the indirect measurement. However, as noticed in Ref. [16], the effective quantum efficiency can be increased at will by squeezing the probe mode b in the direction of the quadrature $\hat{b}_\phi$. More precisely, the probe is prepared in the squeezed vacuum

$$|v(\phi)\rangle \equiv \hat{S}_\phi|0\rangle, \qquad \hat{S}_\phi = \exp\left(-\frac{r}{2}\, e^{2i\phi} b^{\dagger 2} - \text{H.c.}\right), \tag{15}$$

where $r > 0$ denotes the squeezing parameter. One has

$$\hat{S}_\phi \hat{b}_\phi \hat{S}_\phi^\dagger = e^r \hat{b}_\phi, \qquad \hat{S}_\phi^\dagger |x\rangle_\phi = e^{r/2}|e^r x\rangle_\phi, \tag{16}$$

with $|x\rangle_\phi \equiv e^{ib^\dagger b\phi}|x\rangle_0$ denoting the eigenvector of $\hat{b}_\phi$ for eigenvalue x. With the help of transformations (15) and (16) it is easy to check that the kernel $\Xi_\varepsilon(x)$ in Eq. (14) coincides with $K_\eta(x)$ in Eq. (8) with an effective quantum efficiency

$$\eta \equiv \frac{e^{2r}\varepsilon}{e^{2r}\varepsilon + 1 - \varepsilon}. \tag{17}$$

Therefore, by increasing the squeezing parameter r it is possible to enhance the effective quantum efficiency η beyond the bound η_*. At this point it may appear that by squeezing the vacuum of b one can consider weaker and weaker interactions $\varepsilon \to 0$, with the possibility of performing repeated measurements on the same system, and, at the same time, with vanishing perturbation at each measurement. However, as we will see immediately, this is not the case, because the squeezing needed to keep η constant amplifies the perturbation back to a finite extent. To demonstrate this, we need to analyze the $\varepsilon \to 0$ limiting behavior of the operator in Eq. (3)

$$\hat{\Omega}(x,\phi) = \varepsilon_\phi^{1/4}\langle \varepsilon^{1/2}x|\hat{S}_\phi e^{\kappa(a_r b^\dagger - a_r^\dagger b)}|0\rangle, \tag{18}$$

where $a_r = \cosh r a + e^{2i\phi}\sinh r a^\dagger$, the powers of ε account for the quadrature rescaling in Eq. (13), and one should keep in mind that the matrix element is evaluated between vectors of the Hilbert space of mode b only. The explicit form of the operator $\hat{\Omega}(x,\phi)$ in Eq. (18) can be evaluated using Eq. (16), and ordering the interaction operator normally with respect to b. After some algebra one obtains

$$\hat{\Omega}(x,\phi) = \left(\frac{2e^{2r}\varepsilon}{\pi}\right)^{1/4} \exp[-(e^{2r}\varepsilon x - e^{-i\phi}\tan\kappa a_r)^2] \times \exp\left(\frac{1}{2}\, e^{-2i\phi}\tan^2\kappa a_r^2\right)|\cos\kappa|^{a_r^\dagger a_r}, \tag{19}$$

which, in the limit of vanishing transmission coefficient $\varepsilon \to 0$ and infinite squeezing parameter $r \to \infty$ at constant quantum efficiency η in Eq. (17), has the asymptotic form

$$\hat{\Omega}(x,\phi) = \left[\frac{2\eta}{\pi(1-\eta)}\right]^{1/4} \exp\left[-\frac{\eta}{1-\eta}(x - \hat{a}_\phi)^2\right]. \tag{20}$$

Equation (20) is the typical form of a von Neumann "reduction" of the state (state projection for $\eta \to 1$). Therefore, we conclude that although the interaction has been taken vanishingly small, the state is in fact "reduced" at each measuring step.

This work was supported by the Office of Naval Research.

*Permanent address: Dipartimento di fisica "A. Volta," via Bassi 6, I 27100 Pavia, Italy.

[1] O. Alter and Y. Yamamoto, Phys. Rev. Lett. **74**, 4106 (1995).

[2] Y. Aharonov, J. Anandan, and L. Vaidman, Phys. Rev. A **47**, 4616 (1993); Y. Aharonov and L. Vaidman, Phys. Lett. A **178**, 38 (1993). See also the Comment, W. G. Unruh, Phys. Rev. A **50**, 882 (1994).

[3] M. Ueda and M. Kitagawa, Phys. Rev. Lett. **68**, 3424 (1992).

[4] A. Imamoglu, Phys. Rev. A **47**, R4577 (1993).

[5] A. Royer, Phys. Rev. Lett. **73**, 913 (1994); Phys. Rev. Lett. **74**, 1040 (1995).

[6] D. T. Smithey, M. Beck, M. G. Raymer, and A. Faridani, Phys. Rev. Lett. **70**, 1244 (1993).

[7] G. M. D'Ariano, C. Macchiavello, and M. G. A. Paris, Phys. Rev. A **50**, 4298 (1994).

[8] G. M. D'Ariano, J. Eur. Opt. Soc. B (to be published).

[9] G. M. D'Ariano, U. Leonhardt, and H. Paul, Phys. Rev. A **52**, 1801 (1995).

[10] W. K. Wootters and W. H. Zurek, Nature (London) **299**, 802 (1982). In this reference it is shown that the cloning machine violates the superposition principle, which applies to a minimum total number of *three* states, and hence does not rule out the possibility of cloning *two* nonorthogonal states. It is a violation of unitarity that makes cloning any *two* nonorthogonal states impossible [11].

[11] H. P. Yuen, Phys. Lett. A **113**, 405 (1986).

[12] C. W. Helstrom, *Quantum Detection and Estimation Theory* (Academic Press, New York, 1976).

[13] M. Ozawa, in *Squeezed States and Nonclassical Light*, edited by P. Tombesi and E. R. Pike (Plenum, New York, 1989), p. 263.

[14] Such machine is a "state duplicator," and can be achieved by a unitary transformation (see Ref. [11]).

[15] The same conclusion can be reached for any linear interaction between the system and the probe (nonlinear interactions may not preserve the spectrum of the measured operator).

[16] G. M. D'Ariano, S. Mancini, and P. Tombesi (unpublished).