Experimental implementation of unambiguous quantum reading

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We provide the optimal strategy for the unambiguous quantum reading of optical memories, namely when perfect retrieving of information is achieved probabilistically, for the case where noise and loss are negligible. We describe the experimental quantum optical implementations, and provide experimental results for the single photon case.

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I. INTRODUCTION

In the engineering of optical memories (such as CDs or DVDs) and readers, a tradeoff among several parameters must be taken into account. High precision in the retrieving of information is surely an indefeasible assumption, but also energy requirements, size, and weight can play a very relevant role for applications. Clearly, the size and weight of the device increase with the energy, and using a low-energetic radiation to read information reduces the heating of the physical bit, thus allowing for a smaller implementation of the bit itself. Moreover, many physical media (e.g., superconducting devices) dramatically change their optical property if the energy flow overcomes a critical threshold.

In the problem of quantum reading [1–5] of optical devices one's task is to exploit the quantum properties of light to retrieve some classical digital information stored in the optical properties of a given media, making use of as little energy as possible. The quantum reading of optical memories was first introduced in [1]. A realistic model of digital memory was considered, where each cell was composed of a beamsplitter with two possible reflectivities. A single optical input was available to the reading device, while the other one introduced thermal noise in the reading process, so that the problem considered was the discrimination of two lossy and thermal Gaussian channels. It was shown that, for a fixed mean number of photons irradiated over each memory cell, even in the presence of noise and loss, a quantum source of light can retrieve more information than any classical source; in particular in the regime of few photons and high reflectivities. This provided the first evidence that the use of quantum light can provide great improvements in applications in the technology of digital memories such as CDs or DVDs.

In practical implementations noise can sometimes be noticeably reduced [6]. On the other hand, in general, loss inherently affects quantum optical setups. Nevertheless, a theoretical analysis of the ideal (i.e., lossless and noiseless) quantum reading provides a theoretical insight of the problem and a meaningful benchmark for any experimental realization. In this hypothesis the quantum reading of optical devices can be recasted to a discrimination among optical devices with low energy and high precision.

In the ideal reading of a classical bit of information from an optical memory, namely in the discrimination of a quantum optical device from a set of two, different scenarios can be distinguished. A possibility is the on-the-fly retrieving of information (e.g., multimedia streaming), where the requirement is that the reading operation is performed fast; namely, only once, but a modest amount of errors in the retrieved information is tolerable. This scenario corresponds to the problem of minimum energy ambiguous discrimination of optical devices [7–9], where one guesses the unknown device and the task is to minimize the probability of making an error.

On the other hand, in a situation of the criticality of errors and very reliable technology, the perfect retrieving of information is an issue. Then the unambiguous discrimination of optical devices [10], where one allows for an inconclusive outcome (while, in case of conclusive outcome, the probability of error is zero) becomes interesting.

In [2] an optimal strategy for the first scenario (namely, the minimum energy ambiguous discrimination of optical devices) has been provided for the ideal case. This strategy, which exploits the fundamental properties of the quantum theory such as entanglement, allows for the ambiguous discrimination of beamsplitters with the probability of error under any given threshold, while minimizing the energy requirement. The proposed optimal strategy has been compared with a coherent strategy, reminiscent of the one implemented in common CD readers, showing that the former saves orders of magnitude of energy if compared with the latter, and moreover allows for perfect discrimination with finite energy.

In this paper we first extend the results of [2] to the case of unambiguous ideal quantum reading; namely, the minimum energy unambiguous discrimination of optical devices. We provide the optimal strategy for the unambiguous discrimination of beamsplitters with the probability of failure under a given threshold, while minimizing the energy requirement. We show that the optimal strategy does not require any ancillary mode, while in the presence of noise and loss ancillary states improve the performance of the quantum reading setup [1,11]. Both strategies for ambiguous and unambiguous quantum reading reduce to the same optimal strategy for perfect discrimination if the probability of error (in the former case) or the probability of failure (in the latter case) is set to zero. Then we present some experimental setups implementing such optimal strategies which are feasible with present day quantum optical technology, in terms of preparations of single-photon input states, linear optics, and photodetectors. Finally, we will notice that in the experimental implementation of perfect discrimination the noise is negligible.

There are only a few papers reporting on the experimental implementation of discrimination of quantum devices. The authors of [12] dealt with perfect discrimination between single-bit unitary operations using a sequential scheme. In [13], the authors demonstrated the unambiguous discrimination of nonorthogonal processes employing entanglement. In the present paper we report on an experimental realization of the perfect quantum-process discrimination optimized with respect to the minimal energy flux through the unknown device.

The paper is organized as follows. First, in Sec. II we provide general results for the unambiguous discrimination of optical devices (Sec. II A) and the optimal strategy for the unambiguous discrimination of beamsplitters (Sec. II B). Then in Sec. III we describe some experimental setups for the optimal ambiguous (Sec. III A), unambiguous (Sec. III B), and perfect (Sec. III C) discrimination of beamsplitters. Then, in Sec. IV, we provide the results of the experimental implementation of the setup for perfect discrimination. Finally, Sec. V is devoted to conclusions and future perspectives.

II. UNAMBIGUOUS QUANTUM READING

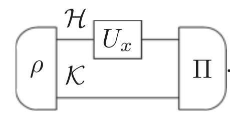
An M-modes quantum optical device [14] is described by a unitary operator U relating M input optical modes with annihilation operators a_i on $\mathcal{H}_i$, to M output optical modes with annihilation operators a_i' on $\mathcal{H}_{i'}$, where $\mathcal{H}_i$ denotes the Fock space of the optical mode i. We denote the total Fock space as $\mathcal{H} = \bigotimes_i \mathcal{H}_i$.

An optical device is called *linear* if the operators of the output modes are related to the operator of the input modes by a linear transformation, namely

$$\begin{pmatrix} \mathbf{a}' \\ \mathbf{a}'^{\dagger} \end{pmatrix} = S \begin{pmatrix} \mathbf{a} \\ \mathbf{a}^{\dagger} \end{pmatrix}, \qquad S := \begin{pmatrix} A & B \\ \bar{B} & \bar{A} \end{pmatrix}, \tag{1}$$

where S is called the scattering matrix, $\bar{X}$ denotes the complex conjugate of X, $\mathbf{a} = (a_1, \dots, a_N)$ is the vector of the annihilation operators of the input mode, and analogously $\mathbf{a}'$ for the output modes. If $B = 0$ in Eq. (1) the device is called *passive* and conserves the total number of photons, that is, $\langle\psi| N |\psi\rangle = \langle\psi| U^\dagger N U |\psi\rangle$ with $N := \sum_i a_i^\dagger a_i$ the *number operator* on $\mathcal{H}$. In the following, for any pure state $|\psi\rangle$, we denote with $\psi := |\psi\rangle\langle\psi|$ the corresponding projector. For any Fock space $\mathcal{H}$, we denote with $|n\rangle$ a Fock basis in $\mathcal{H}$ ($|0\rangle$ denotes the state of the vacuum).

Suppose we want to discriminate between two linear optical passive devices U_1 and U_2. If a single use of the unknown device is available, the most general strategy consists of preparing a bipartite input state $\rho \in \mathcal{B}(\mathcal{H} \otimes \mathcal{K})$ ($\mathcal{K}$ is an ancillary Fock space with mode operators b_i), applying locally the unknown device and performing a bipartite positive operator valued measure (POVM) Π on the output state $(\mathcal{U}_x \otimes \mathcal{I}_\mathcal{K})\rho = (U_x \otimes I_\mathcal{K})\rho(U_x^\dagger \otimes I_\mathcal{K})$ (x can be either 1 or 2)



. (2)

The choice of Π in Eq. (2) depends on the figure of merit taken into account. For example, for ambiguous discrimination $\Pi = \{\Pi_1, \Pi_2\}$ and one's task is to minimize the probability of error

$$P_E(\rho, U_1, U_2) := \mathrm{Tr}[(\mathcal{U}_1 \otimes \mathcal{I}_\mathcal{H})(\rho)\Pi_2 + (\mathcal{U}_2 \otimes \mathcal{I}_\mathcal{H})(\rho)\Pi_1],$$

with $0 \leqslant P_E(\rho, U_1, U_2) \leqslant 1/2$. When $p_1 = p_2 = 1/2$ the minimal probability of error has been proven to be given by the following function [15] of ρ,

$$P_E(\rho^*, U_1, U_2) = \tfrac{1}{2}\{1 - ||[(\mathcal{U}_1 - \mathcal{U}_2) \otimes \mathcal{I}_\mathcal{K}]\,\rho||_1\}, \tag{3}$$

where $||X||_1 = \mathrm{Tr}[\sqrt{X^\dagger X}]$ denotes the trace norm.

For unambiguous discrimination $\Pi = \{\Pi_1, \Pi_2, \Pi_I\}$, $\mathrm{Tr}[(\mathcal{U}_1 \otimes \mathcal{I}_\mathcal{H})(\rho)\Pi_2] = \mathrm{Tr}[(\mathcal{U}_2 \otimes \mathcal{I}_\mathcal{H})(\rho)\Pi_1] = 0$ and one's task is to minimize the probability of inconclusive outcome (failure)

$$P_F(\rho, U_1, U_2) := \mathrm{Tr}[(\mathcal{U}_1 \otimes \mathcal{I}_\mathcal{H} + \mathcal{U}_2 \otimes \mathcal{I}_\mathcal{H})(\rho)\Pi_I], \tag{4}$$

with $0 \leqslant P_F(\rho, U_1, U_2) \leqslant 1$.

Upon denoting with $E_D(\rho) := \mathrm{Tr}[\rho(N \otimes I_\mathcal{K})]$ the energy that flows through the unknown device, the total energy of the input state is $E(\rho) := E_D + \mathrm{Tr}[\rho(I_\mathcal{H} \otimes N_\mathcal{K})]$.

We introduce now the ambiguous (unambiguous) quantum reading problem. For any set of two optical devices $\{U_1, U_2\}$ and any threshold q in the probability of error (failure), find the minimum energy input state ρ^* that allows us to ambiguously (unambiguously) discriminate between U_1 and U_2 with the probability of error (failure) not greater than q, namely

$$\rho^* = \arg \min_{\rho\ \mathrm{s.t.}\ P(\rho, U) \leqslant q} E(\rho), \tag{5}$$

where $P(\rho, U) = P_E(\rho, U)$ for the ambiguous discrimination problem and $P(\rho, U) = P_F(\rho, U)$ for the unambiguous discrimination problem.

A. Unambiguous quantum reading of optical devices

The problem in Eq. (5) was already solved in [2] for the case of ambiguous discrimination. Here we generalize the results obtained in [2] to the unambiguous discrimination problem.

First, notice that for any POVM Π we have $P_F((\mathcal{U}_1 \otimes \mathcal{I}_\mathcal{K})\rho, I, U_2 U_1^\dagger) = P_F(\rho, U_1, U_2)$ and $E((\mathcal{U}_1 \otimes I_\mathcal{K})\rho) = E(\rho)$, so we can restrict our analysis to the case in which $U_1 = I$ and $U_2 = U$, and identify $P_F(\rho, I, U) = P_F(\rho, U)$.

Then notice that, without loss of generality, the constraint in Eq. (5) can be restated as $P_F(\rho, U) = q$. Indeed, for any POVM Π we have that $P_F(\rho, U)$ is a continuous function, and that $P_F(|0\rangle\langle 0|, U) = 1$. So for any ρ with $P_F(\rho, U) < q$ there exists a $0 < \alpha \leqslant 1$ such that $P_F((1-\alpha)\rho + \alpha|0\rangle\langle 0|, U) = q$. Since $E((1-\alpha)\rho + \alpha|0\rangle\langle 0|) < E(\rho)$, the constraint in Eq. (5) becomes $P_F(\rho, U) = q$.

Proposition 1 (Optimal state is pure). For any optical device U and any threshold q in the probability of failure $P_F(\rho, U)$, there exists a state ρ^* which minimizes Eq. (5) such that ρ^* is pure.

Proof. Notice that Eq. (5) is equivalent to $C(\rho,U) := pP_F(\rho,U) + (1-p)E(\rho)$, for any fixed value of p. If ρ^* is the state that minimizes $C(\rho,U)$, for $q := P(\rho^*,U)$ we have that $E(\rho^*)$ gives the minimum possible value for the energy. Since $P_F(\rho,U)$ and $E(\rho)$ are linear functions of ρ, it follows that $C(\rho,U)$ is a linear function of ρ and its minimum is attained on the boundary of its domain, namely for a pure state $|\psi^*\rangle$. ■

As a consequence of Proposition 1, Eq. (5) can be restated as

$$\psi^* = \arg \min_{\psi \text{s.t.} P(\psi,U)=q} E(\psi). \tag{6}$$

For pure states, the probability of failure in the unambiguous discrimination when $p_1 = p_2 = 1/2$ given by Eq. (4) has been proved to be given by [10]

$$P_F(\psi^*,U) = |\langle\psi|\, U\, |\psi\rangle|. \tag{7}$$

Proposition 2 (No ancillary modes are required). For any optical device U and any threshold q in the probability of failure $P_F(\rho,U)$, there exists a state ρ^* which minimizes Eq. (5) such that $\rho^* \in \mathcal{H}$.

Proof. Any pure input state can be written as $|\psi\rangle = \sum_i c_i\, |i\rangle\, |\chi_i\rangle$ where $|i\rangle$ is an orthonormal basis in $\mathcal{H}$ and $|\chi_i\rangle$ are normalized states in $\mathcal{K}$. If we define $|\psi'\rangle := \sum_i c_i\, |i\rangle\, |0\rangle$, it follows that $P_F(\psi,U) = P_F(\psi',U)$ while $E(\psi) \geqslant E(\psi')$, which prooves the statement. ■

Since no ancillary modes are required, the energy $E_D(\psi)$ that flows through the unknown device is equal to the total energy of the input state $E(\rho)$, so minimizing the former instead of the latter [namely, replacing $E(\psi)$ with $E_D(\psi)$ in Eq. (6)] does not change the optimal state.

Notice that the generalization of the results in [2] provided here basically depends on some common properties of the probability of error in Eq. (3) (for ambiguous discrimination) and the probability of failure in Eq. (4) (for unambiguous discrimination), namely the linearity in ρ, the equalities $P_E(|0\rangle\,\langle 0|) = P_F(|0\rangle\,\langle 0|) = 0$ [Eq. (4)], and the monotonicity in $|\langle\psi|\, U\, |\psi\rangle|$ [Eq. (7)].

B. Unambiguous quantum reading of beamsplitters

A beamsplitter is a two-mode linear passive quantum optical device such that $A \in \mathrm{SU}(2)$ in Eq. (1). In the following we will use the basis $\{|n,m\rangle\}$ with respect to which A is diagonal with eigenvalues $e^{\pm i\delta}$, $0 \leqslant \delta \leqslant \pi$. With this choice, for any $|\psi\rangle = \sum_{n,m=0}^{\infty} \alpha_{n,m}\, |n,m\rangle$, we have $U\, |n,m\rangle = e^{i\delta(n-m)}\, |n,m\rangle$, so that $\langle\psi|\, U\, |\psi\rangle = \sum_{n,m=0}^{\infty} |\alpha_{n,m}|^2 e^{i\delta(n-m)}$ and $\langle\psi|\, N\, |\psi\rangle = \sum_{n,m=0}^{\infty} |\alpha_{n,m}|^2(n+m)$. We notice that both these expressions only depend on the squared modulus of the coefficients $\alpha_{n,m}$, so we can assume $\alpha_{n,m}$ to be real and positive.

Here $\lfloor x\rfloor$ ($\lceil x\rceil$) denotes the maximum (minimum) integer number smaller (greater) than x.

Proposition 3 (Unambiguous quantum reading of beamsplitters). For any beamsplitter U and for any threshold q in the probability of failure, there exists a state ψ^* which minimizes Eq. (6) such that

$$|\psi^*\rangle = \frac{1}{\sqrt{2}}\alpha(|0,n^*\rangle + |n^*,0\rangle) + \beta\, |00\rangle\,, \tag{8}$$

where $|\alpha| = \sqrt{\frac{1-q}{1-\cos(\delta n_1)}}$, $|\beta| = \sqrt{1-|\alpha|^2}$, $n^* = \arg\min_{\lfloor x^*\rfloor,\lceil x^*\rceil} E(\psi^*)$, and $x^* = \min[x \in \mathbb{R}^+ | \delta x = \tan(\delta x/2)]$.

Proof. First we prove that the optimal state in Eq. (6) is a superposition of NOON states. For any state $|\psi\rangle = \sum_{n,m} \alpha_{n,m} |n,m\rangle$, the state $|\psi'\rangle = \sqrt{1/2}\sum_l \alpha'_l(|l,0\rangle + |0,l\rangle)$ with $|\alpha'_l|^2 = \sum_{|n-m|=l} |\alpha_{nm}|^2$ is such that

$$\begin{aligned} \langle\psi'|N|\psi'\rangle &= \sum_{n,m=0}^{\infty} \alpha_{nm}^2 |n-m| \leqslant \langle\psi|\, N\, |\psi\rangle\,, \\ |\langle\psi'|U|\psi'\rangle| &= \left|\sum_{n,m=0}^{\infty} \alpha_{nm}^2 \cos(\delta|n-m|)\right| \leqslant |\langle\psi|\, U\, |\psi\rangle|. \end{aligned} \tag{9}$$

So we have $\langle\psi|\, U\, |\psi\rangle \in \mathbb{R}$ and the constraint in Eq. (6) becomes $\langle\psi|\, U\, |\psi\rangle = q$.

Then we prove that the optimal state is the superposition of two NOON states. Let $|\psi^*\rangle = \sqrt{1/2}\sum_n \alpha_n^*(|n,0\rangle + |0,n\rangle)$ be the optimal state and let the set $\{\alpha_n^*\}$ have $N \geqslant 3$ not-null elements. Then there exist n_1 and n_2 such that $\alpha_{n_1},\alpha_{n_2} \neq 0$ and $\cos(\delta n_1) \leqslant q \leqslant \cos(\delta n_2)$. Define $|\chi\rangle := 1/\sqrt{2}\sum_{i=1,2}\beta_{n_i}(|n_i,0\rangle + |0,n_i\rangle)$ such that $\langle\chi|\, U\, |\chi\rangle = q$, and $|\xi\rangle := 1/\sqrt{2}(1-\epsilon)^{-1/2}\sum_n \gamma_n(|n,0\rangle + |0,n\rangle)$, where

$$\gamma_n = \begin{cases} \alpha_n & \text{if } n \neq n_1,n_2 \\ \sqrt{\alpha_n^2 - \epsilon\beta_n^2} & \text{if } n = n_1,n_2, \end{cases}$$

and $\epsilon \leqslant \min(\alpha_{n_1}/\beta_{n_1},\alpha_{n_2}/\beta_{n_2})$. Notice that $\langle\xi|\, U\, |\xi\rangle = q$, and $\langle\psi^*|\, N\, |\psi^*\rangle = \epsilon\,\langle\chi|\, N\, |\chi\rangle + (1-\epsilon)\,\langle\xi|\, N\, |\xi\rangle$. If $\langle\chi|\, N\, |\chi\rangle = \langle\psi^*|\, N\, |\psi^*\rangle$ the statement follows with $|\psi\rangle = |\chi\rangle$. If $\langle\chi|\, N\, |\chi\rangle \neq \langle\psi^*|\, N\, |\psi^*\rangle$, either $\langle\chi|\, N\, |\chi\rangle < \langle\psi^*|\, N\, |\psi^*\rangle$ or $\langle\xi|\, N\, |\xi\rangle < \langle\psi^*|\, N\, |\psi^*\rangle$, that contradicts the hypothesis that $|\psi^*\rangle$ is the optimal state.

Finally, we prove that the optimal state is the superposition of a NOON state and the vacuum. Let $|\psi^*\rangle = 1/\sqrt{2}\sum_{i=1,2}\alpha_{n_i}(|n_i,0\rangle + |0,n_i\rangle)$. Then

$$\langle\psi^*|N|\psi^*\rangle = \frac{n_2\cos(\delta n_1) - n_1\cos(\delta n_2) + q(n_1 - n_2)}{\cos(\delta n_1) - \cos(\delta n_2)}.$$

It is easy to verify (in [2] a proof of this fact is provided) that it is not restrictive to set $n_2 = 0$, so one has $\langle\psi^*|\, N\, |\psi^*\rangle = (1-q)n[1-\cos(\delta n_1)]^{-1}$. Then one can see that it is not restrictive to choose $\pi/2 \leqslant \delta n_1 \leqslant \pi$, where $\langle\psi^*|\, N\, |\psi^*\rangle$ can be proven to be a convex function that attains its minimum for $n_1 = \lfloor x^*\rfloor,\lceil x^*\rceil$, with $x^* = \min[x \in \mathbb{R}^+|\delta x = \tan(\delta x/2)]$. The statement immediately follows. ■

Notice that from Proposition 3 it immediately follows that unambiguous discrimination between beamsplitters U and I can be achieved only if the threshold q in the probability of failure $P_F(\rho,U)$ satisfies the inequality $q \geqslant \cos(\delta n^*)$ (an analogous inequality, namely $K(q) \geqslant \cos(\delta n^*)$ with $K(q) = \sqrt{4q(1-q)}$, holds in the case of ambiguous quantum reading addressed in [2]).

In [2], the optimal coherent strategy (namely, a strategy making use of coherent input states and homodyne measurements) for ambiguous quantum reading is provided. A comparison between the optimal strategy and the optimal coherent strategy showed that the former requires much less energy than the latter when the same threshold in the probability of error is allowed. Here we notice that, since the support of coherent states is the entire Fock space, no measurement exists projecting on its orthogonal complement. For this reason, no coherent strategy exists for the unambiguous discrimination of optical devices.

III. EXPERIMENTAL SETUP FOR QUANTUM READING

In this section we provide experimental setups for ambiguous, unambiguous, and perfect quantum reading, which are feasible with the present quantum optical technology. The input is a single-photon state that can be realized through spontaneous parametric down-conversion or through the attenuation of a laser beam. The evolution is given by a circuit of beamsplitters, one of which is the unknown one, and the final measurement is implemented through photodetectors.

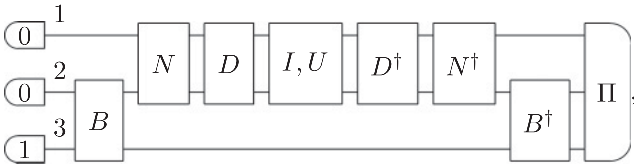
In Proposition 2 we prove that, for the unambiguous quantum reading of optical devices, no ancillary modes are required. The same result has been proved for the case of ambiguous quantum reading in [2]. Nevertheless, the proposed setups for quantum reading make use of three-modes input states; namely, an ancillary mode is employed. This choice is due to the requirement to have an input state with a fixed number of photons to be able to take into account loss. For this reason, our setup minimizes the energy $E_D(\rho)$ that flows through the unknown device, while the total energy of the input state is fixed.

In the following, for any beamsplitter X we denote with A_X the A matrix of X in Eq. (1), so we write

$$A_X = \begin{pmatrix} r_X & -t_X \\ t_X & r_X \end{pmatrix}, \qquad A_X^\dagger = \begin{pmatrix} r_X & t_X \\ -t_X & r_X \end{pmatrix}.$$

We define the reflectivity R_X and the transmittivity T_X of X as $R_X := |r_X|^2$ and $T_X = |t_X|^2$, respectively, with $R_X + T_X = 1$.

The general setup is given by a Mach-Zehnder interferometer with beamsplitters B and $B^\dagger$, acting on modes 2 and 3. In one of the harms of the interferometer (corresponding to mode 2), the following beamsplitters are inserted



where N is a 50:50 beamsplitter, I,U is the unknown beamsplitter, and D is the beamsplitter diagonalizing U. The POVM Π is different for the ambiguous and unambiguous quantum reading.

It is easy to verify that the composition of beamsplitters DN reduces to a phase shifter on mode 2, namely

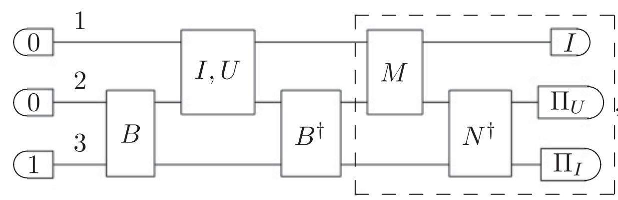
$$A_D = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}, \qquad A_D A_N = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}. \tag{10}$$

It is easy to check that this phase shifter is irrelevant, so in the following we will disregard it.

A. Experimental setup for ambiguous quantum reading of beamsplitters

The optimal strategy for the ambiguous quantum reading of beamsplitters has been provided in [2]. Here we describe an experimental setup implementing such strategy, namely the ambiguous discrimination of a beamsplitter randomly chosen from the set $\{I,U\}$ with equal prior probabilities, with probability of error $P_E(\rho,U)$ under a given threshold q and minimal energy flow trough the unknown device. In the following we set $K(q) := \sqrt{4q(1-q)}$. According to [2] to have $P_E(\rho,U) \leqslant q$, we must have $K(q) \geqslant \sqrt{R_U}$.

The experimental setup is then given by



where the reflectivities and transmittivities of beamsplitters B, M, and $N^\dagger$ are given by

$$R_B = \frac{K(q) - r_U}{1 - r_U}, \qquad T_B = \frac{1 - K(q)}{1 - r_U},$$

$$R_M = \frac{[1 - K(q)][K(q) - r_U]}{(1-2q)^2},$$

$$T_M = \frac{[1 - K(q)](1 + r_U)}{(1-2q)^2},$$

$$R_N = \sqrt{1-q}, \qquad T_N = \sqrt{q}.$$

The optimal measurement for ambiguous discrimination [15] is implemented by the two beamsplitters M and $N^\dagger$ and by the two photocounters Π_U and Π_I surrounded by the dashed line (no measurement is performed on output mode 1). The conditional probabilities $p_{X|Y}$ of detecting a photon in photodetector Π_X given that the unknown device is Y are given by

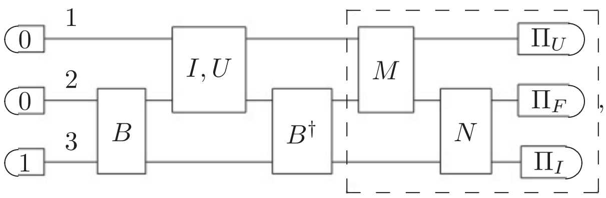
$$p_{U|U} = p_{I|I} = 1 - q, \qquad p_{I|U} = p_{U|I} = q.$$

Detecting a photon in Π_U or Π_I implies that the unknown beamsplitter is U or I, respectively, with probability of error q.

B. Experimental setup for unambiguous quantum reading of beamsplitters

We provided the optimal strategy for the unambiguous quantum reading of beamsplitters in Proposition 3. Here we describe an experimental setup implementing such a strategy, namely the unambiguous discrimination of a beamsplitter randomly chosen from the set $\{I,U\}$ with equal prior probabilities, with the probability of failure $P_F(\rho,U)$ under a given threshold q and minimal energy flow trough the unknown device. According to Proposition 3 to have $P_F(\rho,U) \leqslant q$, we must have $q \geqslant \sqrt{R_U}$.

The experimental setup is given by



where the reflectivities and transmittivities of beamsplitters B, M, and N are given by

$$R_B = \frac{q - r_U}{1 - r_U}, \qquad T_B = \frac{1-q}{1-r_U},$$
$$R_M = \frac{(\sqrt{1+r_U} - \sqrt{q(q-r_U)})^2}{(1+q)^2},$$
$$T_M = \frac{(\sqrt{q(1+r_U)} + \sqrt{q-r_U})^2}{(1+q)^2},$$
$$R_N = \sqrt{1-q}, \qquad T_N = \sqrt{q}.$$

The optimal measurement for unambiguous discrimination [10] is implemented by the two beamsplitters M and N and by the three photocounters Π_U, Π_I, and Π_F surrounded by the dashed line. The conditional probabilities $p_{X|Y}$ of detecting a photon in photodetector Π_X given that the unknown device is Y are given by

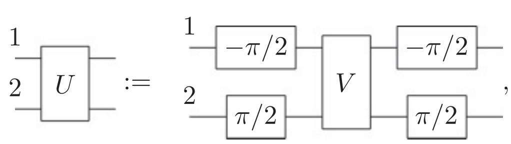
$$p_{U|U} = p_{I|I} = 1 - q, \quad p_{I|U} = p_{U|I} = 0,$$
$$p_{F|U} = p_{F|I} = q.$$

Detecting a photon in Π_U or Π_I implies that the unknown beamsplitter is certainly U or I, respectively, while detecting a photon in Π_F declares a failure with probability q.

C. Experimental setup for perfect quantum reading of beamsplitters

The optimal strategies for the ambiguous and unambiguous quantum reading of beamsplitters of [2] and Proposition 3 reduce to the same optimal strategy for perfect quantum reading of beamsplitters when the threshold q in the probability of error (for the ambiguous case) and failure (for the ambiguous case) is set to zero.

In this case, the condition $r_U \leqslant 0$ given by Proposition 3 can be satisfied upon defining U as the composition of a beamsplitter V with $r_V \geqslant 0$ with $\pm\pi/2$ phase shifters on its input and output modes, according to



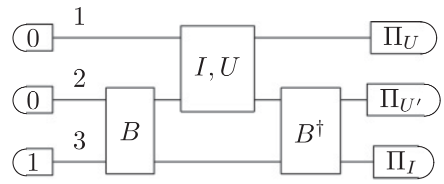
(11)

and it is easy to verify that

$$A_U = \begin{pmatrix} e^{-i\frac{\pi}{2}} & 0 \\ 0 & e^{i\frac{\pi}{2}} \end{pmatrix} A_V \begin{pmatrix} e^{-i\frac{\pi}{2}} & 0 \\ 0 & e^{i\frac{\pi}{2}} \end{pmatrix} = \begin{pmatrix} -r_V & -t_V \\ t_V & -r_V \end{pmatrix},$$

with $0 \leqslant r_V, t_V \leqslant 1$, so that $r_U \leqslant 0$ and perfect discrimination is indeed possible.

The experimental setup is then given by

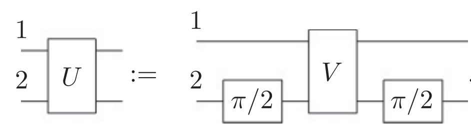


(12)

where the reflectivity and transmittivity of beamsplitter B are given by

$$R_B = \frac{r_V}{1 + r_V}, \qquad T_B = \frac{1}{1 + r_V}.$$

Notice that the two $-\pi/2$ phase shifters on mode 1 in Eq. (11) are irrelevant and can be discarded since the one on the input mode acts on the vacuum and the one on the output mode is immediately followed by a photodetector, so we can redefine



(13)

The optimal measurement for perfect discrimination is implemented by the three photocounters Π_U, $\Pi_{U'}$, and Π_I. The conditional probabilities $p_{X|Y}$ of detecting a photon in photodetector Π_X given that the unknown device is Y are given by

$$p_{U|U} = 1 - r_V, \qquad p_{U'|U} = r_V, \qquad p_{I|I} = 1,$$
$$p_{I|U} = p_{U|I} = p_{U'|I} = 0.$$

Detecting a photon in Π_U or $\Pi_{U'}$ implies that the unknown beamsplitter is certainly U, while detecting a photon in Π_I implies that the unknown device is certainly I.

IV. EXPERIMENTAL IMPLEMENTATION OF QUANTUM READING

To demonstrate the experimental feasibility of quantum reading we have built a laboratory setup for perfect discrimination of two beam splitters according to scheme (12) (see Fig. 1). It consists of a Mach-Zehnder interferometer (MZI) with an additional beamsplitter in its upper arm. This additional beamsplitter has a variable splitting ratio and it serves as an unknown device to be discriminated.

We use a heralded single photon source based on spontaneous parametric down-conversion (SPDC). Namely, we employ a collinear frequency-degenerate SPDC process with type-II phase matching in a 2-mm-long beta barium borate (BBO) crystal pumped by a cw laser diode (Coherent Cube) at 405 nm. In this process pairs of photons at 810 nm are created. Photons from each pair are separated by a polarizing beamsplitter and coupled into single-mode optical fibers. One

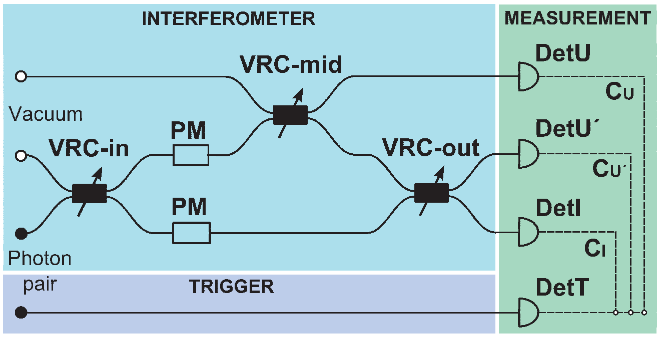


FIG. 1. (Color online) Scheme of the experiment. VRC: variable ratio couplers, PM: phase modulators, Det: detectors, C: coincidence electronics.

of them is led directly to trigger detector DetT which heralds the creation of a pair (Perkin-Elmer SPCM AQR-14FC, dark counts 180 s^{-1}, total efficiency cca 50%). The other one enters MZI through the variable ratio coupler VRC-in.

An additional variable ratio coupler, VRC-mid, represents an unknown device. When its reflectivity equals 1 it corresponds to device I. To switch to device U one has to set required splitting ratio and apply additional phase shift, see scheme (13). For practical reasons we apply, without the loss of generality, a cumulative phase shift in front of the beamsplitter. In the experiment the phase shifts are introduced by electrooptical phase modulators (PM) manufactured by EO Space. Their half-wave voltages are about 1.5 V. These phase modulators exhibit relatively high dispersion. Therefore one PM is placed in each interferometer arm to compensate dispersion effects. In the case of device U we use the PM in the upper interferometer arm to apply the additional phase shift of π.

The output fibers from the unknown device and from the interferometer are led to detectors DetU, DetU′, and DetI. These detectors are parts of the Perkin-Elmer quad module SPCM-AQ4C (dark counts 370–440 s^{-1}, total efficiencies about 50%).

TABLE I. Results for device I. R_v: reflectivities of devices U; $p_{U|I},p_{U'|I},p_{I|I}$: theoretical probabilities of photon detection at detectors DetU, DetU′, DetI, respectively, $f_{U|I},f_{U'|I},f_{I|I}$: relative frequencies measured at detectors DetU, DetU′, DetI, respectively (measured in coincidence with DetT).

R_v	$p_{U\|I}$	$p_{U'\|I}$	$p_{I\|I}$	$f_{U\|I}$	$f_{U'\|I}$	$f_{I\|I}$
0.0	0	0	1	0.000	0.002	0.998
0.1	0	0	1	0.000	0.012	0.988
0.2	0	0	1	0.000	0.018	0.982
0.3	0	0	1	0.000	0.012	0.988
0.4	0	0	1	0.000	0.023	0.977
0.5	0	0	1	0.000	0.022	0.978
0.6	0	0	1	0.000	0.014	0.986
0.7	0	0	1	0.000	0.011	0.989
0.8	0	0	1	0.000	0.013	0.987
0.9	0	0	1	0.000	0.018	0.982
1.0	0	0	1	0.000	0.021	0.979

TABLE II. Results for devices U. R_v: reflectivity of device U; $p_{U|U},p_{U'|U},p_{I|U}$: theoretical probabilities of photon detection at detectors DetU, DetU′, DetI, respectively, $f_{U|U},f_{U'|U},f_{I|U}$: relative frequencies measured at detectors DetU, DetU′, DetI, respectively (measured in coincidence with DetT).

R_v	$p_{U\|U}$	$p_{U'\|U}$	$p_{I\|U}$	$f_{U\|U}$	$f_{U'\|U}$	$f_{I\|U}$
0.0	1.000	0.000	0	0.986	0.000	0.014
0.1	0.684	0.316	0	0.680	0.295	0.025
0.2	0.553	0.447	0	0.551	0.440	0.009
0.3	0.452	0.548	0	0.455	0.542	0.003
0.4	0.368	0.633	0	0.369	0.623	0.008
0.5	0.293	0.707	0	0.288	0.691	0.021
0.6	0.225	0.775	0	0.219	0.758	0.022
0.7	0.163	0.837	0	0.160	0.830	0.010
0.8	0.106	0.894	0	0.100	0.891	0.009
0.9	0.051	0.949	0	0.046	0.946	0.007
1.0	0.000	1.000	0	0.000	0.980	0.020

To reduce the effect of the phase drift caused by fluctuations of temperature and temperature gradients we apply both passive and active stabilization. The experimental setup is covered by a shield minimizing air flux around the components. Besides, after every 3 seconds of measurement an active stabilization is performed. It measures intensities for phase shifts 0 and $\pi/2$ and if necessary it calculates phase compensation and applies corrective voltage to the phase modulator in the lower interferometer arm. This results in the precision of the phase setting during the measurement period better than $\pi/200$.

For each pair of devices U and I the proper splitting ratio of fiber couplers VRC-in and VRC-out must be set to discriminate these devices optimally. We have made measurements for 11 different devices U with intensity reflectances $0,0.1,0.2,\ldots,1$. For each pair of devices U and I the counts at detectors DetU, DetU′, and DetI were cumulated during 30 3-s measurement intervals interlaced by stabilization procedures. All measurements were done in coincidence with the trigger detector

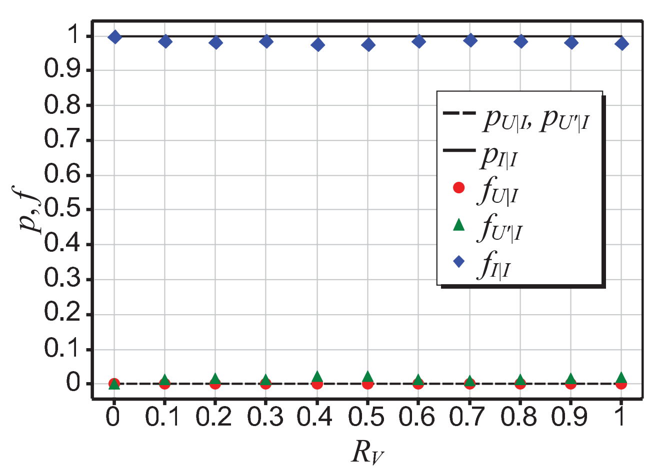


FIG. 2. (Color online) Results for device I: Detection probabilities and measured relative frequencies as functions of reflectivity R_v. Different reflectivities correspond to different devices U (VRC-mid).

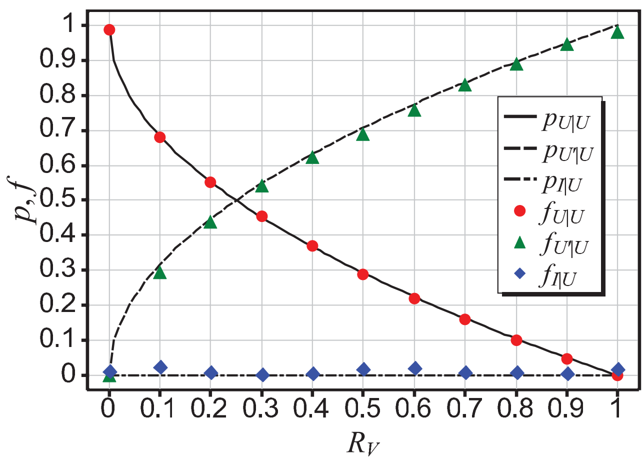


FIG. 3. (Color online) Results for devices U: Detection probabilities and measured relative frequencies as functions of reflectivity R_v. Different reflectivities correspond to different devices U (VRC-mid).

DetT. It means we measured coincidence counts $C_U, C_{U'}, C_I$ between detectors DetT-DetU, DetT-DetU′, and DetT-DetI, respectively, using a 3-ns coincidence time window. These results were normalized to obtain relative frequencies, $f_j = C_j/(C_U + C_{U'} + C_I), j = U, U', I$, which can be compared with the theoretical probabilities of detection.

The measured relative frequencies and theoretical probabilities are listed in Tables I and II and shown in Figs. 2 and 3, respectively. Table I and Fig. 3 show the results obtained with device I inserted, Table II and Fig. 3 summarize the results for devices U. Each row in the tables corresponds to one pair of I and U with R_v being the reflectivity of device U. One can observe very good agreement between the theory and experiment. Small discrepancies appear mainly due to imperfections in splitting-ratio settings, phase fluctuations, and polarization misalignment. In coincidence measurements the contribution of detector noise is completely negligible.

V. CONCLUSION

In this paper we considered the unambiguous quantum reading of optical memories, on the assumption that noise and loss are negligible. In Sec. II we showed that the optimal strategy for the unambiguous discrimination of optical devices can be derived by extending the results proved for the ambiguous case.

In Sec. III we presented some experimental implementations of quantum reading for both the ambiguous and the unambiguous cases. In the proposed setups the input state is fixed to be a single photon state. By making use of an ancillary mode it was possible to tune the amount of energy flowing through the device.

Finally, in Sec. IV we provide experimental results for the perfect quantum reading. The advantage of the implemented setup is that in an ideal case there is exactly one photon at the output ports. It makes detection relatively easy. Nevertheless, it is still a superposition of a single photon and vacuum that is entering the unknown device. So the unknown device is exposed just to a fraction of the energy of a single photon on average. Even if the overall probability of the success of the setup was relatively low because of technological losses, we were able to measure precisely the relative probabilities of all outputs and our experiments convincingly validated the predictions of the exposed theory.

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