Local observables for entanglement detection of depolarized states

G. Mauro D'Ariano,* Chiara Macchiavello,† and Matteo G. A. Paris‡

Quantum Optics and Information Group, Istituto Nazionale di Fisica della Materia, Unità di Pavia,§ I-27100 Pavia, Italy
and Dipartimento di Fisica "A. Volta," Università di Pavia, I-27100 Pavia, Italy

We present an explicit construction of entanglement witnesses for depolarized states in an arbitrary finite dimension. For an infinite dimension, we generalize the construction to twin beams perturbed by Gaussian noises in the phase and in the amplitude of the field. We show that entanglement detection for all these families of states require only three local measurements. The explicit form of the corresponding set of local observables needed for an entanglement witness is derived.

I. INTRODUCTION

Entanglement plays an essential role in almost all aspects of quantum information theory [1]. Entangled states are the key ingredients of many quantum protocols such as quantum teleportation, quantum dense coding, and entanglement-based quantum cryptography. However, entanglement can be in general corrupted by interaction with the environment. Therefore, entangled states that are available for experiments are usually mixed states, and it becomes crucial to establish whether or not entanglement has survived the environmental noise.

The issue of experimental entanglement detection was first addressed for pure states in Ref. [2]. More recently, in Ref. [3], procedures based on the use of collective measurements were proposed. Later, in Ref. [4] a general method to detect entanglement with few local measurements was presented and optimal schemes were designed for two-dimensional systems, bound entangled states, and entangled states of three qubits. In Ref. [5], a method for local detection of nonseparable states has been derived for bipartite states in dimension d and to some families of states of n qubits; it was shown, in particular, that in the bipartite case and for d as a prime number the method achieves the lower bound of $d+1$ measurements derived in Ref. [4]. In this paper, we extend the approach of Ref. [4] to depolarized bipartite states in an arbitrary dimension, and show how entanglement can be efficiently detected by identifying the minimal needed set of local observables, the so-called *quorum* of observables. Moreover, we address the problem of entanglement detection for continuous variables (CV) and find entanglement witnesses (EW) for a twin-beam state (TWB) corrupted by Gaussian noises, both in the phase and in the amplitude of the field. In this case, efficient homodyne-tomographic procedures are analyzed suited to local detection of entanglement. We found that for all the families of states that we have considered, a rank-4 witness operator is sufficient to detect entanglement. Notice that this

result is not in contradiction with the ones derived in Ref. [5] because we assume to have more knowledge about the family of states.

The paper is organized as follows. In Sec. II, we construct the EW for bipartite depolarized entangled states in an arbitrary finite dimension, and give the explicit form of the corresponding local quorum. In Sec. III, we analyze the case of bipartite CV systems. In particular, we study the family of twin-beam states corrupted by Gaussian noise, both in the phase and in the amplitude of the field, and show how to detect entanglement by employing homodyne-tomographic techniques. In Sec. IV, we close the paper with a summary of the results and final comments.

II. DEPOLARIZED STATES IN ARBITRARY DIMENSION

In this section, we will show how to detect entanglement locally for depolarized states in an arbitrary finite dimension d, namely, for the family of states

$$\rho = p|\psi\rangle\langle\psi| + \frac{1-p}{d^2} I \otimes I, \tag{1}$$

where $|\psi\rangle$ is any bipartite entangled normalized pure state of systems with dimension d, I is the $d \times d$ identity operator and $0 \leq p \leq 1$. If $|\psi\rangle$ is a maximally entangled state, the states in Eq. (1) coincide with the family of the so-called isotropic states.

We will now introduce a more convenient notation. Given a basis $\{|i\rangle \otimes |j\rangle\}$ for the Hilbert space $\mathcal{H}_1 \otimes \mathcal{H}_2$ (with $\mathcal{H}_1$ and $\mathcal{H}_2$ generally not isomorphic), we can write any vector $|\Psi\rangle\rangle \in \mathcal{H}_1 \otimes \mathcal{H}_2$ as

$$|\Psi\rangle\rangle = \sum_{ij} \Psi_{ij} |i\rangle_1 \otimes |j\rangle_2. \tag{2}$$

The above notation [6] exploits the correspondence between states $|\Psi\rangle\rangle$ in $\mathcal{H}_1 \otimes \mathcal{H}_2$ and Hilbert-Schmidt operators $\Psi = \Sigma_{ij}\Psi_{ij}|i\rangle\langle j|$ from $\mathcal{H}_1$ to $\mathcal{H}_2$. The following relations are an immediate consequence of definition (2):

$$A \otimes B |\Psi\rangle\rangle = |A\Psi B^T\rangle\rangle, \tag{3}$$

$$\langle\langle A|B\rangle\rangle = \text{Tr}[A^\dagger B], \tag{4}$$

*Electronic address: dariano@unipv.it
†Electronic address: chiara@unipv.it
‡Electronic address: paris@unipv.it
§URL: http://www.qubit.it

where B^T denotes the transposition of the operator B with respect to the chosen basis $\{|i\rangle\}$. As mentioned above, in the following we will consider only bipartite states on $\mathcal{H} \otimes \mathcal{H}$, where $\mathcal{H}$ has dimension d.

In this notation, the depolarized state (1) takes the form

$$R = p|\Psi\rangle\rangle\langle\langle\Psi| + \frac{1-p}{d^2} I \otimes I. \tag{5}$$

Let us briefly recall the definition of EW [7,8]. A state ρ is entangled iff there exists an Hermitian operator W such that $\text{Tr}[W\rho] < 0$, while $\text{Tr}[W\rho_{sep}] \geq 0$ for all separable states ρ_{sep}. The operator W is called *entanglement witness*. For entangled states with nonpositive partial transpose (NPT), W can be explicitly constructed as $W = (|\epsilon\rangle\langle\epsilon|)^\theta$, where O^θ denotes the partial transposition of O on the second Hilbert space, and $|\epsilon\rangle$ is the eigenvector of ρ^θ that corresponds to the minimum eigenvalue [8]. Notice that this is not the only method to construct entanglement witnesses. Other techniques, working for both NPT and (positive partial transpose) PPT entangled states, have been suggested, as for example, in Refs. [9,10].

The entangled states of the form (1) have nonpositive partial transpose [11,12]. Following the approach of Ref. [4], we will show how to detect entangled states within the family (1) by explicitly deriving EW according to the above construction.

The partial transpose of the state R can be written as

$$R^\theta = p(\Psi \otimes I)E(\Psi^\dagger \otimes I) + \frac{1-p}{d^2} I \otimes I, \tag{6}$$

where E is the swap operator, i.e., $E = \Sigma_{ij}|i\rangle\langle j| \otimes |j\rangle\langle i|$.

As mentioned above, in order to construct a witness operator for the family of states (1), we look for the eigenvector of R^θ corresponding to the minimum eigenvalue. Therefore, we can start by writing explicitly the eigenvalue equation

$$R^\theta|A\rangle\rangle = \lambda|A\rangle\rangle, \tag{7}$$

where $|A\rangle\rangle$ is the eigenvector for the eigenvalue λ. By using the properties (4) and Eq. (6), we can also write

$$R^\theta|A\rangle\rangle = p|\Psi A^T\Psi^*\rangle\rangle + c|A\rangle\rangle, \tag{8}$$

where $c = (1-p)/d^2$, and O^* denotes complex conjunction of the operator O with respect to the chosen basis $\{|i\rangle\}$. Therefore, the eigenvalue equation in operatorial terms takes the form

$$\lambda A = p\Psi A^T\Psi^* + cA, \tag{9}$$

and can be more conveniently written as

$$\Psi A^T\Psi^* = \mu A, \quad \mu = (\lambda - c)/p. \tag{10}$$

We now use the singular-value decomposition of the matrix Ψ, namely, $\Psi = X\Sigma Y^\dagger$, where X and Y are unitary operators, while Σ is the diagonal operator containing the eigenvalues $\{\sigma_j\}$ of $\sqrt{\Psi\Psi^\dagger}$—the so-called *singular values* of

Ψ—which are conventionally ordered decreasingly. The above equation then takes the form

$$X\Sigma Y^\dagger A^T X^*\Sigma Y^T = \mu A. \tag{11}$$

By multiplying Eq. (11) by $X^\dagger$ on the left and by Y^* on the right, and upon defining

$$B = Y^\dagger A^T X^*, \tag{12}$$

Eq. (11) can be written in the compact form

$$B^T = \mu^{-1}\Sigma B\Sigma. \tag{13}$$

The last equation can be conveniently expressed by explicitly writing its matrix elements as follows:

$$b_{ij} = \mu^{-1}b_{ji}\sigma_i\sigma_j. \tag{14}$$

By reiterating the above equation, one obtains

$$b_{ij} = \mu^{-2}\sigma_i^2\sigma_j^2 b_{ij}, \tag{15}$$

which is fulfilled for

$$\mu^2 = \sigma_i^2\sigma_j^2. \tag{16}$$

For values of i and j that cannot satisfy Eq. (16), we necessarily have $b_{ij} = 0$. We now want to specify the form of operator B corresponding to the minimum eigenvalue λ. Notice first that for eigenvalues $\lambda < c$, the parameter μ is negative, and therefore, according to Eq. (14), all diagonal elements of B vanish. This is the case, in particular, when the minimum eigenvalue λ_m is negative. We will now explicitly derive the form of B corresponding to the minimum eigenvalue λ_m. Assume that σ_1 and σ_2 are the two largest elements of Σ and $\sigma_1 \geq \sigma_2$. Then, from Eq. (10), the minimum eigenvalue λ_m takes the form $\lambda_m = -p\sigma_1\sigma_2 + c$, and according to Eq. (15) the matrix elements of the operator B corresponding to λ_m (which we will denote by $\bar{B}$) are

$$\bar{b}_{12} = -\bar{b}_{21} = 1, \tag{17}$$

while all the other elements vanish. Therefore, the operator $\bar{B}$ has rank 2 and takes the explicit form

$$\bar{B} = \left(\begin{array}{cc|c} 0 & 1 & \\ -1 & 0 & 0 \\ \hline & 0 & 0 \end{array} \right) \tag{18}$$

The expression for the operator A corresponding to the minimum eigenvalue λ_m, which we will call $\bar{A}$, follows from the definition of B in Eq. (12) and is given by

$$\bar{A} = X\bar{B}Y^T. \tag{19}$$

The EW for the family of states (1) can then be derived as

$$\bar{W} = (|\bar{A}\rangle\rangle\langle\langle\bar{A}|)^\theta. \tag{20}$$

Notice that the same form of $\bar{B}$ is also valid for degenerate maximum singular value σ_1, although in this case the solution is not unique. Moreover, an interesting feature of the resulting witness operator is that it has rank 4, independent of the dimension d of the subsystems. We also want to point out that the EW for the states (1) does not depend on the value of p, but only on some *a priori* knowledge about the state $|\Psi\rangle\rangle$, namely, on the singular values of Ψ and on the form of the operators X and Y.

As an illustration we will consider two explicit examples. When $|\Psi\rangle\rangle$ is a maximally entangled state in dimension d of the form $|\Psi\rangle\rangle = (1/\sqrt{d})\Sigma_j|jj\rangle$, i.e., the operator Σ is proportional to the identity, with $\sigma_i = 1/\sqrt{d}$, then the operator $\bar{A}$ corresponding to a state $|\bar{A}\rangle\rangle = (|ij\rangle - |ji\rangle)/\sqrt{2}$ can be used to construct a witness operator. In this case, the state is separable iff $p > 1/(d+1)$.

As a second example, let us consider an initial state with Schmidt number 2, i.e., $\sigma_1 = \sigma_2 = 1/\sqrt{2}$ and $\sigma_i = 0$ for $i > 2$. In this case, the corresponding EW is constructed from $|\bar{A}\rangle\rangle = (|01\rangle - |10\rangle)/\sqrt{2}$, where $|01\rangle$ and $|10\rangle$ are the basis states related to σ_1 and σ_2. The state is entangled when $p \geq 2/(d^2 + 2)$.

We will now show how to detect entanglement for the family of states (1) by measuring only three local observables. The matrix $\bar{A}$ in Eq. (19) can be written as

$$\bar{A} = iX(\sigma_y \oplus \mathbf{0})Y^T, \tag{21}$$

where σ_y is a Pauli matrix (acting between the two levels of the two-dimensional subspace spanned by $\bar{A}$), $\oplus$ denotes the direct sum, and $\mathbf{0}$ is the null matrix. If P is the projection operator over the subspace where $\bar{A}$ is not null, the above expression can be rewritten as

$$\bar{A} = iX'P'\Sigma_y P'Y^*, \tag{22}$$

where $X' = XY^T$, $P' = Y^*PY^T$, and $\Sigma_y = Y^*\sigma_y \oplus \mathbf{0}Y^T$. Inserting the above expression in the definition (20) of $\bar{W} = (\bar{A} \otimes I)E(\bar{A}^\dagger \otimes I)$, we have

$$W = (X'\Sigma_y \otimes I)(E_2 \oplus \mathbf{0})(\Sigma_y X'^\dagger \otimes I), \tag{23}$$

where E_2 is the swap operator for the two-dimensional subspace spanned by the support of $\bar{A}$. Since one has

$$E_2 = \frac{1}{2}\sum_{\alpha = t,x,y,z}\sigma_\alpha \otimes \sigma_\alpha, \tag{24}$$

where $\sigma_t \equiv I$, the EW can be finally written as

$$\bar{W} = \frac{1}{2}I \otimes I + \sum_{\alpha = x,y,z}\frac{1}{2}\tilde{\sigma}_\alpha \otimes \sigma_\alpha, \tag{25}$$

with

$$\tilde{\sigma}_\alpha = X'\Sigma_y \sigma_\alpha \Sigma_y X'^\dagger. \tag{26}$$

As we can see from Eq. (25), the witness operator $\bar{W}$ can be measured by performing the measurements of only three local observables $\tilde{\sigma}_\alpha \otimes \sigma_\alpha$, $\alpha = x,y,z$. This result generalizes that of Ref. [4] to an arbitrary dimension for states of the form (1): in all cases only three local observables are sufficient.

As mentioned in the Introduction, in Ref. [5] a different method to detect entanglement of d-dimensional states has been proposed. This method is valid for states of the form $|\psi\rangle = \Sigma_{k=0}^{d-1}a_k|kk\rangle$ with $a_k \geq 0$ and requires the measurement of $d+1$ observables. Compared to our method, it needs the measurements of a larger number of observables, but, on the other hand, it does not require the knowledge of the values of the coefficients a_k in the density matrix.

III. PERTURBED TWIN BEAM IN CONTINUOUS VARIABLES

In this section, we address the construction and the measurement of EW for CV. At first, we have to define the families of states we are going to consider. These cannot be a trivial generalization of the isotropic states, since both maximally entangled states and the identity are unphysical states in an infinite-dimensional Hilbert space. We start from the "maximally" entangled state of two CV systems at finite energy, which is given by

$$|\Psi\rangle\rangle = \Psi \otimes I|I\rangle\rangle, \quad \Psi = \sqrt{1-|x|^2}\,x^{a^\dagger a}, \quad |x| < 1, \tag{27}$$

where without loss of generality we will consider x as real. Here and in the following, with $a^\dagger, b^\dagger$, and a,b we will denote the creation and annihilation operators of two independent harmonic oscillators, respectively, with commutations $[a,a^\dagger] = [b,b^\dagger] = 1$. For electromagnetic radiation, the harmonic oscillators describe two field modes, and Eq. (27) describes the so-called TWB obtained by parametric down-conversion of the vacuum in a nondegenerate optical parametric amplifier. In this case, $\bar{n} = 2x^2/(1-x^2)$ represents the average number of photons of the TWB. In practice, TWBs are the most reliable source of CV entanglement: indeed, experimental implementation of quantum information protocols such as teleportation, have been obtained using TWB of radiation.

Let us now analyze the family of states that are obtained by perturbing a TWB with a noisy environment. We will consider Gaussian noises both in the phase and in the amplitude of the field modes. Thermal noise is a special case of the present Gaussian displacement noise, whereas the noise coming from the addition of a thermal state has been considered in Ref. [13]. In this case, our results coincide with the ones given there.

The action of a phase-destroying environment on the TWB is described by the master equation

$$\dot{R} = \frac{\gamma}{2}[2a^\dagger a R a^\dagger a - (a^\dagger a)^2 R - R(a^\dagger a)^2 \tag{28}$$

$$+ 2b^\dagger b R b^\dagger b - (b^\dagger b)^2 R - R(b^\dagger b)^2], \tag{29}$$

where $\dot{R}$ denotes the time derivative of the state R. The solution of Eq. (29) for initial condition $R_0 = |\Psi\rangle\rangle\langle\langle\Psi|$ can be expressed as

$$R(t) = (1-x^2)\sum_{p,q} x^{p+q} e^{-\gamma t|p-q|^2} |pp\rangle\langle qq|, \qquad (30)$$

where we used the abbreviate notation $|ij\rangle$ for $|i\rangle \otimes |j\rangle$. The correlations between the modes are reduced in the mixture (30) compared to the initial TWB state. However, as we will see by explicitly constructing an EW, phase noise never leads to a separable state, i.e., the entanglement is not destroyed for any value of γt.

In order to obtain an EW for the family $R(t)$, we construct and diagonalize the partial transpose $R^{\theta}(t)$,

$$R^{\theta}(t) = (1-x^2)\sum_{pq} x^{p+q} e^{-\gamma t|p-q|^2} |pq\rangle\langle qp|. \qquad (31)$$

The eigenvalues equation $R^{\theta}_{\gamma}|\psi\rangle\rangle = \lambda|\psi\rangle\rangle$ is solved by

$$\lambda_n = (1-x^2)x^{2n}, \quad |\psi_n\rangle = |nn\rangle,$$

$$\lambda^{\pm}_{nm} = \pm(1-x^2)x^{n+m} e^{-\gamma t(n-m)^2},$$

$$|\psi^{\pm}_{nm}\rangle\rangle = \frac{1}{\sqrt{2}}(|nm\rangle \pm |mn\rangle). \qquad (32)$$

The minimum eigenvalue is given by $\lambda^-_{01} = -(1-x^2)x e^{-\gamma}$ corresponding to the eigenvector

$$|\psi^-_{01}\rangle\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle). \qquad (33)$$

The eigenvector $|\psi^-_{01}\rangle\rangle$ does not depend on γt, and thus is suitable to build a proper EW for this family of states. We have

$$W = \frac{1}{2}(|\psi^-_{01}\rangle\rangle\langle\langle\psi^-_{01}|)^{\theta}$$

$$= \frac{1}{2}(|01\rangle\langle 01| + |10\rangle\langle 10| - |00\rangle\langle 11| - |11\rangle\langle 00|).$$

$$(34)$$

The expectation value

$$\text{Tr}[R(t)W] = \lambda^-_{01} < 0 \quad \forall t, x \qquad (35)$$

is always negative and thus the state $R(t)$ is never separable, for any value of t, and for any value of the initial TWB parameter x. In other words, the entanglement is decreased, but never destroyed by phase noise. It can also be proved [14] that $R(t)$ can be distilled. The result in Eq. (35) proves the conjecture suggested in Ref. [15], where the entanglement analysis of a phase-perturbed TWB was performed by numerical evaluation of the relative entropy of entanglement.

Let us now consider the family of states obtained by perturbing a TWB state by Gaussian amplitude noise, namely,

$$R_{\kappa} = \mathcal{G}_{\kappa} \otimes \mathcal{G}_{\kappa}(|\Psi\rangle\rangle\langle\langle\Psi|), \qquad (36)$$

where for a single-mode state ρ, the map of the Gaussian noise is given by

$$\mathcal{G}_{\kappa}(\rho) \doteq \int \frac{d^2\alpha}{\pi\kappa} e^{-|\alpha|^2/\kappa} D(\alpha)\rho D^{\dagger}(\alpha), \qquad (37)$$

where $D(\alpha) = \exp\{\alpha a^{\dagger} - \bar{\alpha}a\}$ denoting the displacement operator. We notice that the operator (34) obtained for phase perturbation is an EW also for Gaussian amplitude noise. Omitting positive factors, we have

$$\text{Tr}[R_{\kappa}W] \propto \kappa - 1 + \frac{1}{2}\frac{1-x}{1+x} \overset{\bar{n}\gg 1}{\simeq} \kappa - 1 + \frac{1}{4\bar{n}}. \qquad (38)$$

Equation (38) says that R_{κ} becomes separable if $\kappa \gtrsim 1 - \frac{1}{4}\bar{n}^{-1}$, a result that can be also obtained by a direct check of the positivity of the partial transpose (PPT condition) [16]. The family R_{κ}, in fact, is composed of Gaussian states, for which the PPT condition is necessary and sufficient for separability [17]. It should be mentioned that the constructive procedure suggested in Ref. [8] fails to provide an EW for the the family R_{κ}, in particular, it does not lead to a state-independent witness.

In principle, the EW (34) can be measured by using only three observables, as in the finite-dimensional case. However, there is no feasible implementation of the measuring apparatus corresponding to the quorum in the present CV case. Since we are interested only in the expectation value of W, we could use quantum tomography (for a recent tutorial review on quantum tomography, see Ref. [18]). However, a tomographic determination of W is useful only if one requires a smaller number of observables than those needed for reconstructing the full state. Indeed, this is the case for the EW in Eq. (34). In fact, for two modes of radiation a_1 and a_2, the expectation value $\langle O \rangle \doteq \text{Tr}[RO]$ of a generic operator O can be obtained by *local* repeated measurements of the quadratures $X_{1\phi_1} = \frac{1}{2}(a_1^{\dagger}e^{i\phi_1} + a_1 e^{-i\phi_1})$ and $X_{2\phi_1} = \frac{1}{2}(a_2^{\dagger}e^{i\phi_2} + a_2 e^{-i\phi_2})$ as follows:

$$\langle O \rangle = \int\int \frac{d\phi_1}{\pi}\frac{d\phi_2}{\pi}\langle R[O](X_{1\phi_1},\phi_1;X_{2\phi_2}\phi_2)\rangle, \quad (39)$$

namely, by averaging the over the phases $\phi_{1,2}$ and over an ensemble of repeated measurements the function of the two quadratures $R[O](x_1,\phi_1;x_2,\phi_2)$—so-called *estimator* or *kernel function*—depending on the operator O. The kernel function for Hilbert-Schmidt operators can be obtained directly by means of the trace [18] $R[O](x_1,\phi_1;x_2,\phi_2) = \text{Tr}[R(X_{1\phi_1} - x_1)R(X_{2\phi_2} - x_2)O]$ with $R(x) = -\lim_{\varepsilon\to 0^+}\frac{1}{2}\text{Re}(x+i\varepsilon)^{-2}$. For the operator W in Eq. (34), we have

$$R[W](x_1,\phi_1;x_2,\phi_2)=f_{00}(x_1)f_{11}(x_2)+f_{11}(x_1)f_{00}(x_2)$$
$$-2\cos(\phi_1+\phi_2)f_{01}(x_1)f_{01}(x_2),$$
$$(40)$$

where

$$f_{00}(x)=2\Phi(1,\tfrac{1}{2};-2x^2),$$

$$f_{01}(x)=4\sqrt{\pi}x\Phi(2,\tfrac{3}{2};-2x^2),$$

$$f_{11}(x)=2[\Phi(1,\tfrac{1}{2};-2x^2)-2\Phi(2,\tfrac{1}{2};-2x^2,)] \quad (41)$$

and $\Phi(a,b;z)$ denotes the confluent hypergeometric function. Remarkably, $R[W]$ depends only on the sum of the two phases $\phi_{1,2}$, and shows only a couple of oscillations. Therefore, the number of measurements to detect the entanglement witness is much smaller than that needed to reconstruct just the first few matrix elements of the state, say, in the photon number representation, since the number of oscillations of the estimators for such matrix elements increases linearly with their photon-number index. The precision of the tomographic estimation can be further improved by adaptive techniques [19].

If we are allowed to mix the two modes after perturbation, the characterization of entanglement for the family R_κ can be obtained by measuring a single quadrature. In fact, for Gaussian states, a necessary and sufficient condition to have entanglement after a beam splitter is that the two inputs show squeezing (in mutually orthogonal directions) [20,21]. Therefore, if we impinge the two modes of the perturbed TWB in a beam splitter, and then measure the quadrature $X=\tfrac{1}{2}(a^\dagger$

$+a)$ on the sum mode, we have squeezing if and only if the input state is entangled. Therefore, the fluctuation operator $W=\Delta X^2-1/4=X^2-\langle X\rangle^2-1/4$ is an EW, and its expectation value is of course obtained by measuring the quadrature X. The analysis is valid also when the TWB initial parameter x is complex, in which case the phase of the quadrature to be measured coincides with the phase of x. Obviously, if the mixing of the two modes is not possible, one can always reconstruct the above quadrature locally by quantum tomography.

IV. CONCLUSIONS

In this paper, we have given an explicit construction of EW for depolarized states in an arbitrary finite dimension. For infinite dimensions, i.e., for CV, we have introduced isotropic states as twin beams perturbed by Gaussian noises in the phase or in the amplitude of the field, and we have constructed their respective EW as well. We have shown that in all cases entanglement detection needs only a quorum of three local observables, whose explicit form have been derived. For CV it is possible to use also homodyne tomography efficiently to detect entanglement, without determining the matrix elements of the state.

ACKNOWLEDGMENTS

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