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A fictitious photons method for tomographic imaging

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Abstract

We present a new tomographic method that is suited to imaging from very weak signals with a high resolution apparatus. The method arises from a procedure for measuring the density matrix of the radiation field, and relies on a correspondence between images and trace-class matrices. Accurate image reconstructions are achieved with relatively small numbers of data.

1. Introduction

The essential problem of tomographic imaging is to recover a mass distribution $\mu(x,y)$ in a 2-d slab from a finite collection of one-dimensional projections. The situation is schematically sketched in Fig. 1a where $\mu(x,y)$ describes two circular holes in a uniform background. The tomographic machine, say an X-ray equipment, collects many stripe photos of the sample from various directions ϕ, and then numerically performs a mathematical transformation – the so-called *inverse Radon transform* [1] – in order to reconstruct $\mu(x,y)$ from its radial profiles at different ϕ. The problem which is of interest for us is when the radial profiles are not well resolved digitalized functions, but actually represent the density distribution of random points – as if in our X-ray machine the beam is so weak that radial photos are just the collection of many small spots, each from a single X-ray photon (this situation is sketched in Fig. 1b). It is obvious that this case can be reduced to the previous one by counting all points falling in a predetermined 1-d mesh, and giving radial profiles in form of histograms (this is what actually happens in a real machine, using arrays of photodetectors). However, we want to use the whole available information from each "event" – i.e. the exact 1-d location of each spot – in a way which is independent of any predetermined mesh. In practice, this situation occurs when the signal is so weak and the machine resolution is so high (i.e. the mesh-step is so tiny) that only zero or one photon at most can be collected in each channel. As we will see in this paper, this low-signal/high-resolution case naturally brings the imaging problem into the quantum domain. Here, we have at disposal a technique [3] for measuring the density matrix of the field in terms of homodyne outcomes (the equivalent of our radial spots) for different phases of the local oscillator (the equivalent of our ϕ angle). This technique – ac-

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