Optimization in the design of quantum experiments

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Using entanglement in Quantum Measurements

- Quantum Tomographic Calibration of Quantum Operations and POVM's (GMD)
- Discrimination of Quantum Operations (MFS)
- Transmission of reference frames (MFS)

Essential literature

- **Tomography of quantum operations and POVM's**
  - G. M. D'Ariano, and P. Lo Presti, M. G. A. Paris, *Using entanglement improves precision of quantum measurements* Phys. Rev. Lett. 87 270404 (2001)
  - G. M. D'Ariano and P. Lo Presti, *Imprinting a complete information about a quantum channel on its output state,* Phys. Rev. Lett. **91** 047902-1 (2003)
  - G. M. D'Ariano and P. Lo Presti, *Characterization of Quantum Devices,* Springer, Lecture Notes
- **Discrimination of Quantum Operations and improving precision of quantum measurements**
  - G. M. D'Ariano, and P. Lo Presti, M. G. A. Paris, *Using entanglement improves precision of quantum measurements* Phys. Rev. Lett. 87 270404 (2001)
- **Transmitting frames using entanglement with the multiplicity space**
  - G. Chiribella, G. M. D'Ariano, P. Perinotti, and M. F. Sacchi, *Covariant quantum measurements which maximize the likelihood* Phys. Rev A (submitted)
  - G. Chiribella, G. M. D'Ariano, P. Perinotti, and M. F. Sacchi, *Efficient use of quantum resources for the transmission of a reference frame,* Phys. Rev. Lett. (submitted)

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Entangled States

- Entangled states $|\Psi\rangle\rangle \in \mathsf{H} \otimes \mathsf{H}$

$$|\Psi\rangle\rangle = \sum_{nm} \Psi_{nm} |n\rangle \otimes |m\rangle.$$

- Matrix notation (for fixed reference basis in the two Hilbert spaces):

$$A \otimes B |C\rangle\rangle = |AC\, B^{\mathsf{T}}\rangle\rangle,$$

$$|A\rangle\rangle \doteq \sum_{nm} A_{nm} |n\rangle \otimes |m\rangle \equiv A \otimes I |I\rangle\rangle \equiv I \otimes A^{\mathsf{T}} |I\rangle\rangle\ ,$$

Entangled States

$$|I\rangle\!\rangle = \sum_n |n\rangle \otimes |n\rangle.$$

- Isomorphism $\mathsf{HS}(\mathsf{H}) \simeq \mathsf{H} \otimes \mathsf{H}$ between the Hilbert space $\mathsf{HS}(\mathsf{H})$ of Hilbert-Schmidt operators on H and $\mathsf{H} \otimes \mathsf{H}$

$$\langle\!\langle A|B\rangle\!\rangle \equiv \mathrm{Tr}[A^\dagger B].$$

- Measure of the entanglement for pure states: von Neumann entropy $S(\rho) = -\mathrm{Tr}[\rho \ln \rho]$ of the local state

$$\rho = \mathrm{Tr}_2[|\Psi\rangle\!\rangle\langle\!\langle\Psi|] \equiv \Psi\Psi^\dagger.$$

Quantum Operations

- The most general state (conditioned) evolution in quantum mechanics:

  the "quantum operation" (Kraus)

$$\rho \to \frac{\mathscr{E}(\rho)}{\mathrm{Tr}[\mathscr{E}(\rho)]}.$$

- The **quantum operation** $\mathscr{E}$ is a map on traceclass operators that is

  1. linear
  2. trace-decreasing
  3. completely positive

- The normalization $\mathrm{Tr}[\mathscr{E}(\rho)] \leq 1$ is the probability that the transformation occurs.

Complete positivity

- A map is completely positive if preserves the positivity of any state on which it acts locally, namely, for any state R in an extended Hilbert space $\mathsf{H} \otimes \mathsf{K}$:

$$\mathscr{E} \otimes \mathscr{I}_{\mathsf{K}}(R) \geq 0$$

- Counterexample: the **transposition map** (with respect to some fixed basis)

$$\Theta(\rho) = \rho^{\mathsf{T}}$$

If you consider the singlet state

$$|\Psi\rangle\!\rangle = \frac{1}{\sqrt{2}} \left[| \uparrow\rangle \otimes | \downarrow\rangle - | \downarrow\rangle \otimes | \uparrow\rangle \right].$$

One check that

$$R_{out} = \Theta \otimes \mathscr{I}(|\Psi\rangle\!\rangle\langle\!\langle\Psi|)$$

$$\langle\!\langle\Phi| R_{out} |\Phi\rangle\!\rangle = -\frac{1}{2}$$

for

$$|\Phi\rangle\!\rangle = \frac{1}{\sqrt{2}} \left[\uparrow\rangle \otimes | \uparrow\rangle + | \downarrow\rangle \otimes | \downarrow\rangle \right].$$

Complete positivity

- One-to-one correspondence $\mathscr{E} \leftrightarrow R_{\mathscr{E}}$ between quantum operations on $\mathsf{T}(\mathsf{H})$ and positive operators $R_{\mathscr{E}}$ on $\mathsf{H} \otimes \mathsf{H}$:

$$R_{\mathscr{E}} = \mathscr{E} \otimes \mathscr{I}_{\mathsf{H}}(|I\rangle\!\rangle\langle\!\langle I|) \; ,$$
$$\mathscr{E}(\rho) = \mathrm{Tr}_2[I \otimes \rho^{\mathsf{T}} R_{\mathscr{E}}] \; ,$$

- The most general form for $\mathscr{E}$ is (Kraus)

$$\mathscr{E}(\rho) = \sum_n K_n \rho K_n^\dagger \; ,$$

where the operators K_n satisfy the bound

$$\sum_n K_n^\dagger K_n \leq I \; .$$

Quantum Operations: examples

1. Unitary transformations:

$$\mathscr{E}(\rho) = U\rho U^\dagger.$$

2. Pure operations:

$$\mathscr{E}(\rho) = A\rho A^\dagger \ ,$$

   A contraction, i. e. $\|A\| \leq 1$.

3. Mixing transformations:

$$\mathscr{E}(\rho) = \sum_n K_n \rho K_n^\dagger.$$

4. Deterministic transformations (channels):

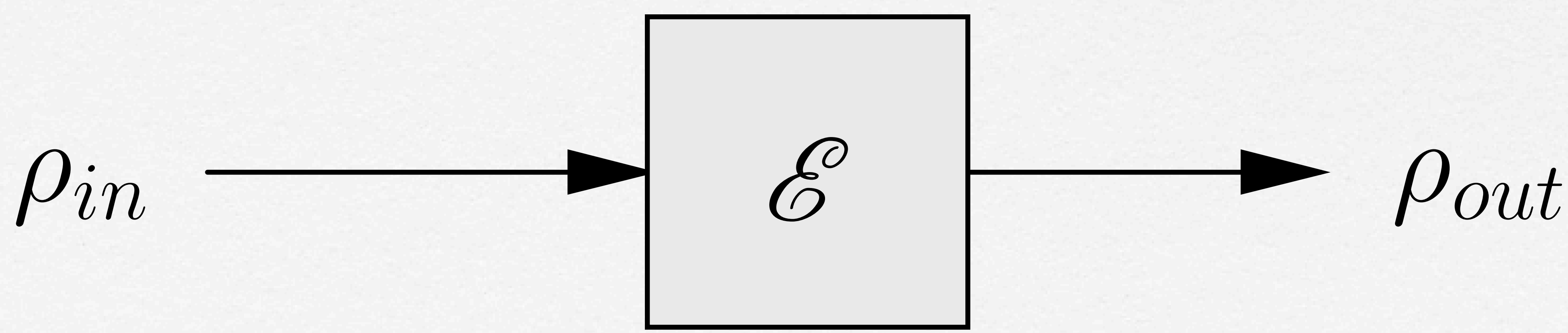
$$\mathrm{Tr}[\mathscr{E}(\rho)] = \mathrm{Tr}[\rho] \Rightarrow \sum_n K_n^\dagger K_n = I \ .$$

How to characterize the QO of a device

- Any linear device (e.g. optical lens, amplifier) can be completely described by a **transfer matrix** which gives the output vector by matrix-multiplying the input vector.

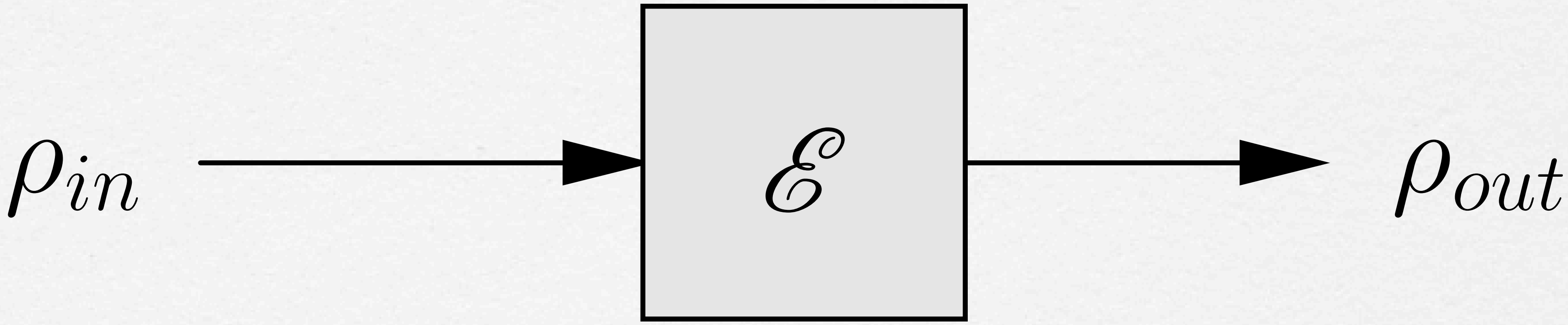
- Problem: **how to reconstruct the full transfer matrix of a device?**

- Answer (brute force): **by scanning a *basis* of possible inputs**, and measuring the corresponding outputs.



How to characterize the QO of a device

- In quantum mechanics the inputs and outputs are density operators, and the role of the transfer matrix is played by the quantum operation of the device (which is linear a part from a normalization).

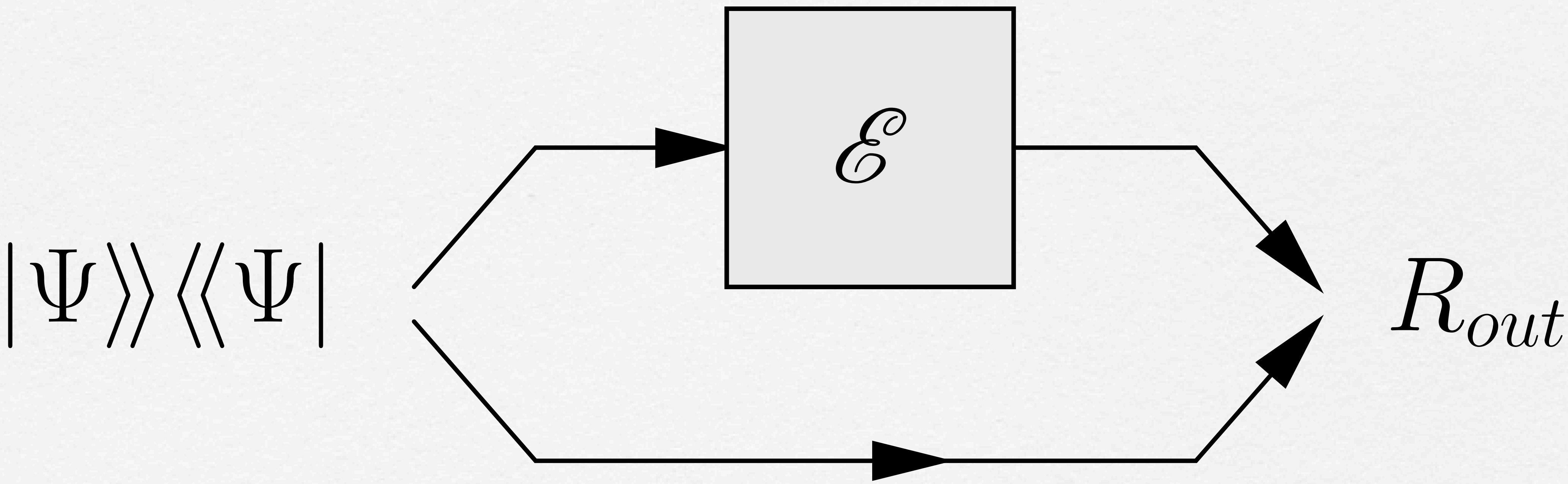


- We need to run a complete orthogonal basis of quantum states $|n\rangle$ ($n = 0, 1, 2, \ldots$), along with their linear combinations $\frac{1}{\sqrt{2}}(|n'\rangle + i^k|n''\rangle)$, with $k = 0, 1, 2, 3$ and i denoting the imaginary unit.

- However, **the availability of a basis of states in the lab is a very hard technological problem**.

The entangled input

- **Quantum parallelism of entanglement: a single entangled input state $|\Psi\rangle\rangle$ is equivalent to scanning all states in parallel.**



- We need to put the entangled state at the input of the device with two identical quantum systems prepared in an entangled state $|\Psi\rangle\rangle$, and only one of the two systems undergoing the quantum operation $\mathscr{E}$, whereas the other is left untouched.

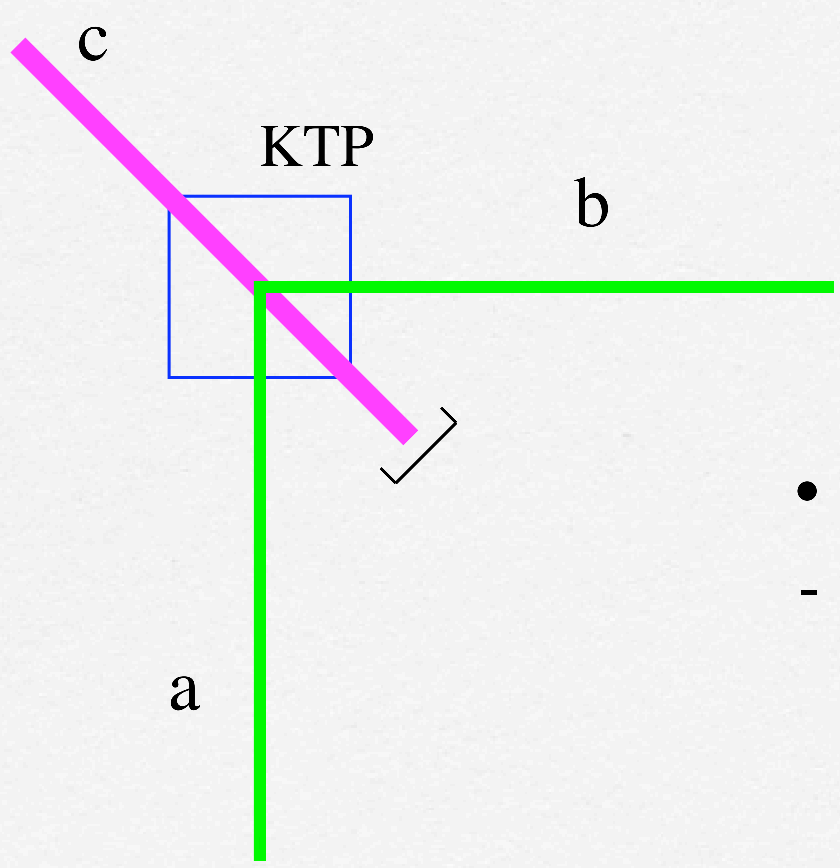
The entangled input

- In tensor-product notation this setup is expressed as the quantum operation
$$R_{out} = \mathscr{E} \otimes \mathscr{I}(|\Psi\rangle\rangle\langle\langle\Psi|).$$
where the entangled state $|\Psi\rangle\rangle$ is given by
$$|\Psi\rangle\rangle = \sum_{nm} \Psi_{nm}|n\rangle \otimes |m\rangle.$$

- For fixed state $|\Psi\rangle\rangle$ (Ψ full-rank) the output state $R_{out} \equiv R_{\mathscr{E}}(\Psi)$ is in one-to-one correspondence with the quantum operation of the device $\mathscr{E}$.

Availability of entangled inputs

- Full-rank entangled states can be easily generated in Quantum Optics from **parametric downconversion of vacuum**



- Hamiltonian $\boxed{H \propto c a^\dagger b^\dagger + h.c.}$ where $\omega_c = \omega_a + \omega_b$.
  - From input vacuum in a and b and classical pump c produces the "twin-beam"

$$\boxed{|\Psi\rangle\!\rangle = (1 - |\xi|^2)^{\frac{1}{2}} \sum_{n=0}^{\infty} \xi^n |n\rangle \otimes |n\rangle}$$

- Faithful entangled states of *qubits* can be generated by means of networks of controlled-NOT gates.

Quantum Tomography

- Howto determine the output state?

  Answer: using quantum tomography.

- Quantum tomography is a method to estimate the ensemble average $\langle H \rangle$ of any arbitrary operator H on H by using only measurement outcomes of a *quorum* of observables $\{O_l\}$.

- The density matrix ρ_{ij} corresponds to estimating the ensemble averages $\langle |i\rangle\langle j| \rangle$.

- This means that any operator H can be expanded as

$$H = \sum_l \langle Q_l, H \rangle O_l,$$

  for suitable scalar product $\langle , \rangle$ and dual set $\{Q_l\}$.

- Hence, the tomographic estimation of the ensemble average $\langle H \rangle$ is obtained as double averaging over both the ensemble and the quorum.

Quantum Tomography

- Very powerful experimental method. General approach for unbiasing the instrumental noise. Improvements based on adaptive techniques, maximum-likelihood strategies, etc.

- For multipartite quantum systems, simply a quorum is the tensor product of single-system quorums: this means that, in our case, we just need to make two local quorum measurements jointly on the two systems.

Pauli Tomography

Pauli matrices with identity I , σ_x , σ_y , σ_z: orthonormal basis for the qubit operator space:

$$H = \tfrac{1}{2}\{\boldsymbol{\sigma} \cdot \mathrm{Tr}[\boldsymbol{\sigma} H] + I\,\mathrm{Tr}[H]\}\ .$$

- Tomographic estimation:

$$\begin{aligned} \langle H \rangle &= \tfrac{1}{3} \sum_{\alpha=x,y,z} \langle E_H(\sigma_\alpha;\alpha)\rangle\ , \\ E_H(\sigma_\alpha;\alpha) &= \tfrac{3}{2}\mathrm{Tr}[H\sigma_\alpha]\sigma_\alpha + \tfrac{1}{2}\mathrm{Tr}[H] \end{aligned}$$

Pauli Tomography

- Qubit realized by polarization of single photon states.

$$\begin{aligned}\sigma_z &= h^\dagger h - v^\dagger v, \\ |\uparrow\rangle &\equiv |1\rangle_h|0\rangle_v, \; |\downarrow\rangle \equiv |0\rangle_h|1\rangle_v,\end{aligned}$$

$$\sigma_y = e^{i\frac{\pi}{4}\sigma_x}\sigma_z e^{-i\frac{\pi}{4}\sigma_x} \;,$$

$$e^{-i\frac{\pi}{4}\sigma_x}|1\rangle_h|0\rangle_v = \frac{1}{\sqrt{2}}[|1\rangle_h|0\rangle_v - i|0\rangle_h|1\rangle_v] \equiv |1\rangle_l|0\rangle_r \;,$$

$$\sigma_x = e^{-i\frac{\pi}{4}\sigma_y}\sigma_z e^{i\frac{\pi}{4}\sigma_y} \;,$$

$$e^{i\frac{\pi}{4}\sigma_y}|1\rangle_h|0\rangle_v = \frac{1}{\sqrt{2}}[|1\rangle_h|0\rangle_v - |0\rangle_h|1\rangle_v] \equiv |1\rangle_{\swarrow\!\!\nearrow}|0\rangle_{\nwarrow\!\!\searrow} \;.$$

Pauli Tomography

$$e^{-i\frac{\pi}{4}\sigma_x}|1\rangle_h|0\rangle_v = \frac{1}{\sqrt{2}}[|1\rangle_h|0\rangle_v - i|0\rangle_h|1\rangle_v] \equiv |1\rangle_l|0\rangle_r \ ,$$

$$\sigma_x = e^{-i\frac{\pi}{4}\sigma_y}\sigma_z e^{i\frac{\pi}{4}\sigma_y} \ ,$$

$$e^{i\frac{\pi}{4}\sigma_y}|1\rangle_h|0\rangle_v = \frac{1}{\sqrt{2}}[|1\rangle_h|0\rangle_v - |0\rangle_h|1\rangle_v] \equiv |1\rangle_{\nearrow}|0\rangle_{\nwarrow} \ .$$

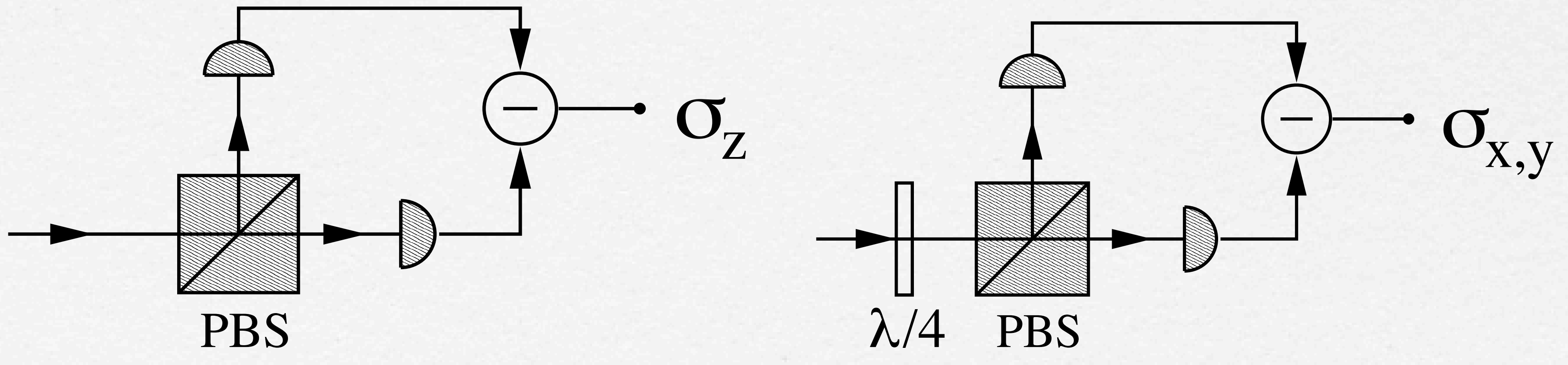


Figure 1: Pauli-matrix detectors for photon-polarization qubits.

Homodyne Tomography

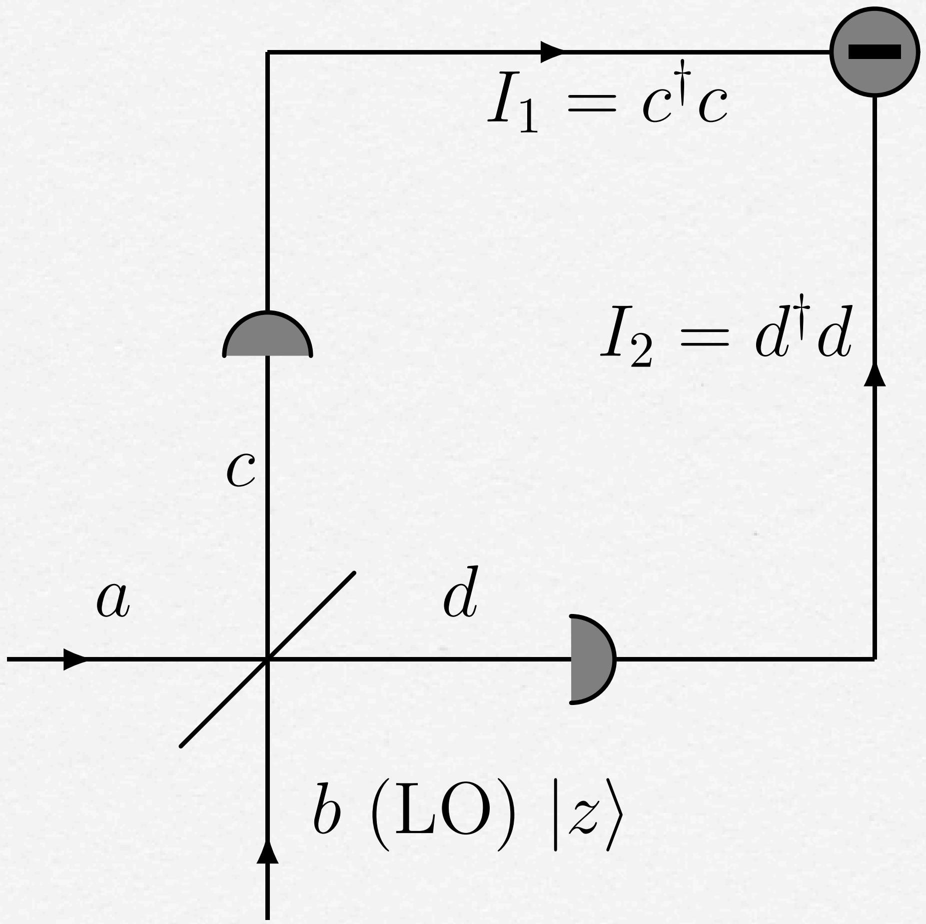
- In quantum optics a *quorum* for each mode of the field is given by the set of *quadratures*

$$X_\phi = \frac{1}{2}\left(a^\dagger e^{i\phi} + a e^{-i\phi}\right).$$

- One has

$$\begin{aligned} \langle H \rangle &= \int_0^\pi \frac{\mathrm{d}\,\phi}{\pi} \langle E_H(X_\phi;\phi)\rangle \, , \\ E_H(x;\phi) &= \frac{1}{4}\int_{-\infty}^{+\infty} \mathrm{d}\,k\,|k| \mathrm{Tr}[H e^{ikX_\phi}] e^{-ikx} \, , \end{aligned}$$

Homodyne detector



$$
\begin{aligned}
I_D &= I_1 - I_2 \\
&= a^\dagger b + b^\dagger a \\
&\simeq 2|z| X_\phi = \tfrac{1}{2}\left(a^\dagger e^{i\phi} + a e^{-i\phi}\right)
\end{aligned}
$$

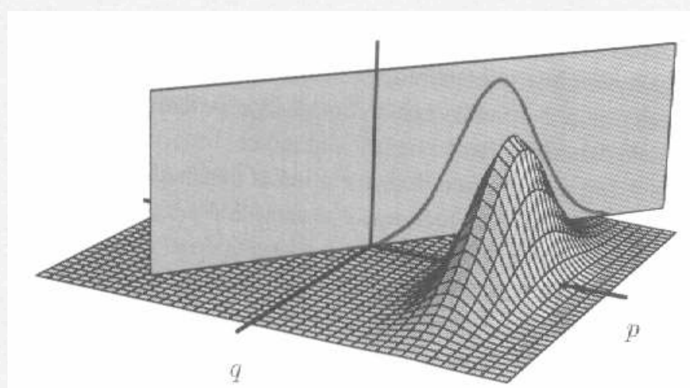
$$
c = \frac{1}{\sqrt{2}}(a+b) \ , \quad d = \frac{1}{\sqrt{2}}(a-b) \ .
$$

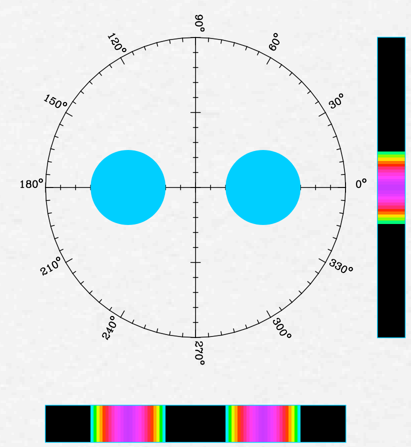
In the *strong LO limit* ($z \to \infty$) a balanced homodyne detector measures the quadrature X_ϕ of the field at any desired phase ϕ with respect to the local oscillator (LO).

Homodyne Tomography

$$\begin{aligned}\langle H\rangle &= \int_0^\pi \frac{\mathrm{d}\,\phi}{\pi}\langle E_H(X_\phi;\phi)\rangle\ ,\\ E_H(x;\phi) &= \frac{1}{4}\int_{-\infty}^{+\infty}\mathrm{d}\,k\,|k|\mathrm{Tr}[He^{ikX_\phi}]e^{-ikx}\ ,\end{aligned}$$

- Analogy with the Radon transform for *imaging*

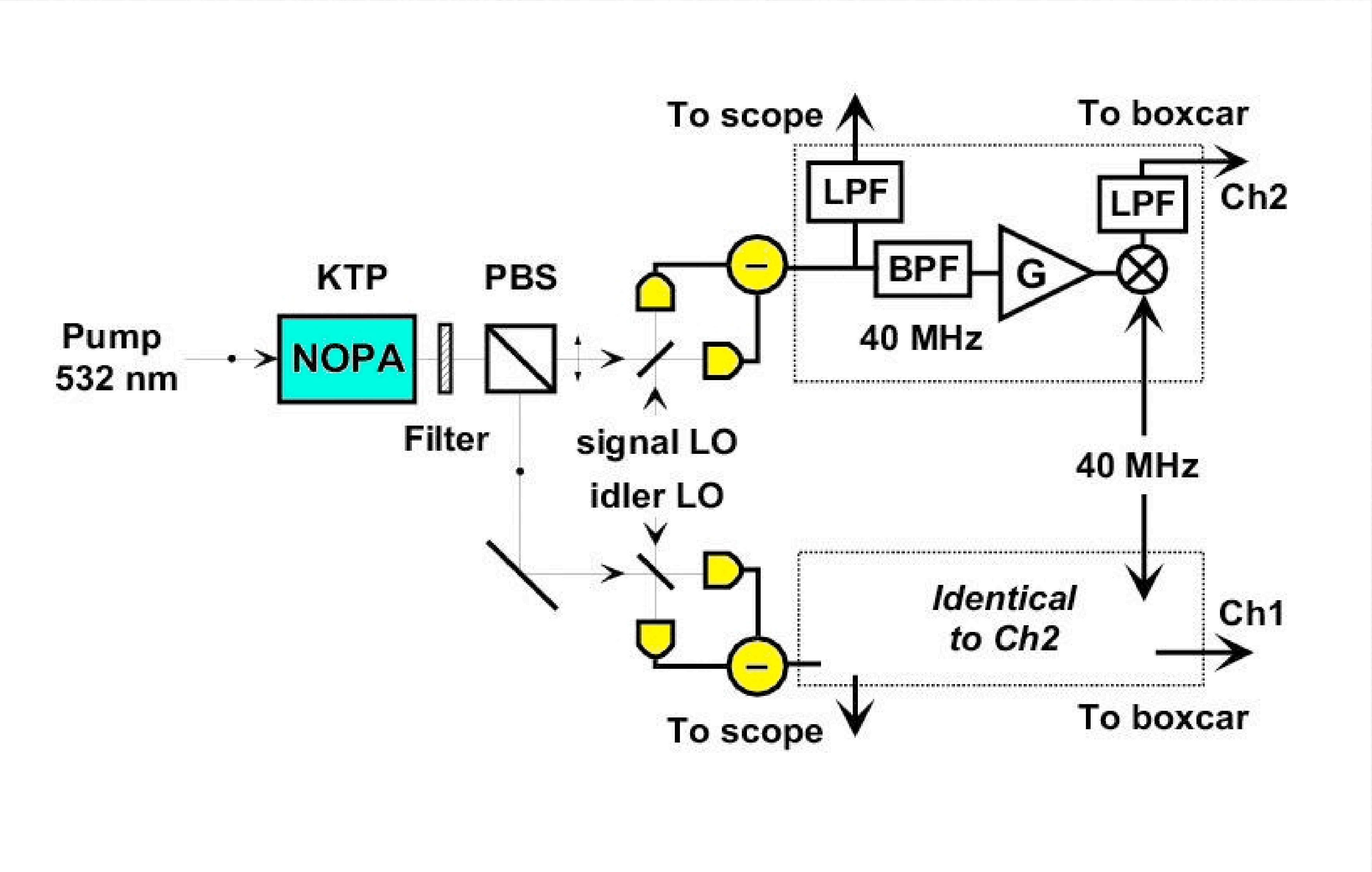




- A **tomography** of a two dimensional image $W(\alpha,\bar{\alpha})$ is a collection of one dimensional projections $p(x;\phi)$ at different values of the observation angle ϕ.

$$W(\alpha,\overline{\alpha}) = \int_{-\infty}^{+\infty}\frac{dr|r|}{4}\int_0^\pi\frac{d\phi}{\pi}\int_{-\infty}^{+\infty}dx\,p(x;\phi)\exp\left[ir(x-\alpha_\phi)\right]\ .$$

Homodyne Tomography

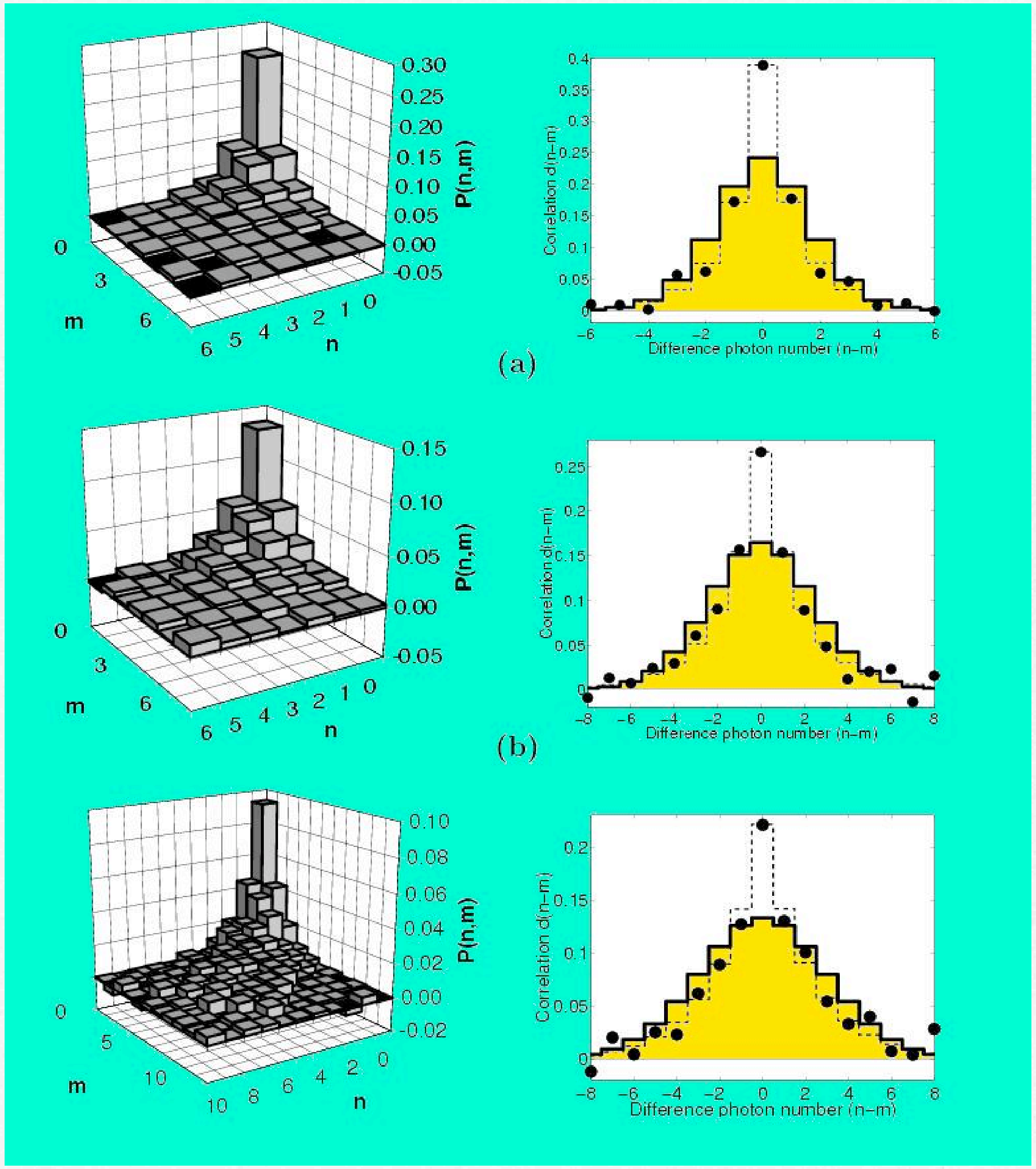


Measurement of the joint photon-number probability distribution of a twin-beam: schematic of the experimental setup. NOPA, non-degenerate optical parametric amplifier; LOs, local oscillators; PBS, polarizing beam splitter; LPFs, low-pass filters; BPF, band-pass filter; G, electronic amplifier. Electronics in the two channels are identical.

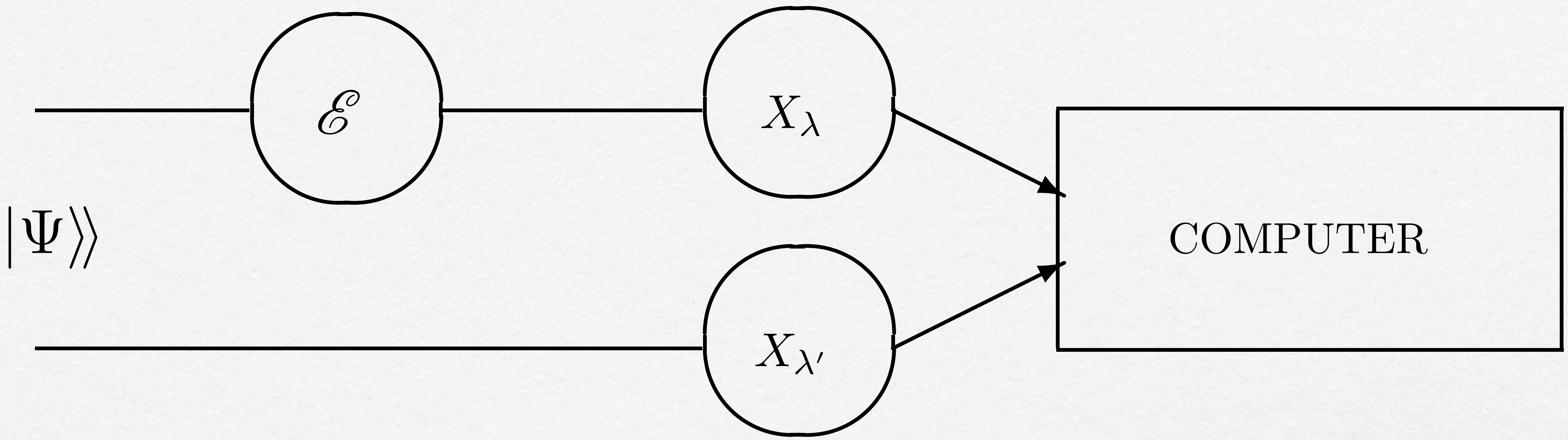
Homodyne Tomography

Results

Left: Measured joint photon-number probability distributions for the twin-beam state. Right: Difference photon number distributions corresponding to the left graphs (filled circles, experimental data; solid lines, theoretical predictions; dashed lines, difference photon-number distributions for two independent coherent states with the same total mean number of photons and $\overline{n} = \overline{m}$.) (a) 400000 samples, $\overline{n} = \overline{m} = 1.5$, $N = 10$; (b) 240000 samples, $\overline{n} = 3.2$, $\overline{m} = 3.0$, $N = 18$; (c) 640000 samples, $\overline{n} = 4.7$, $\overline{m} = 4.6$, $N = 16$. The measured distributions exhibit up to 1.9 dB of quantum correlation between the signal and idler photon numbers.



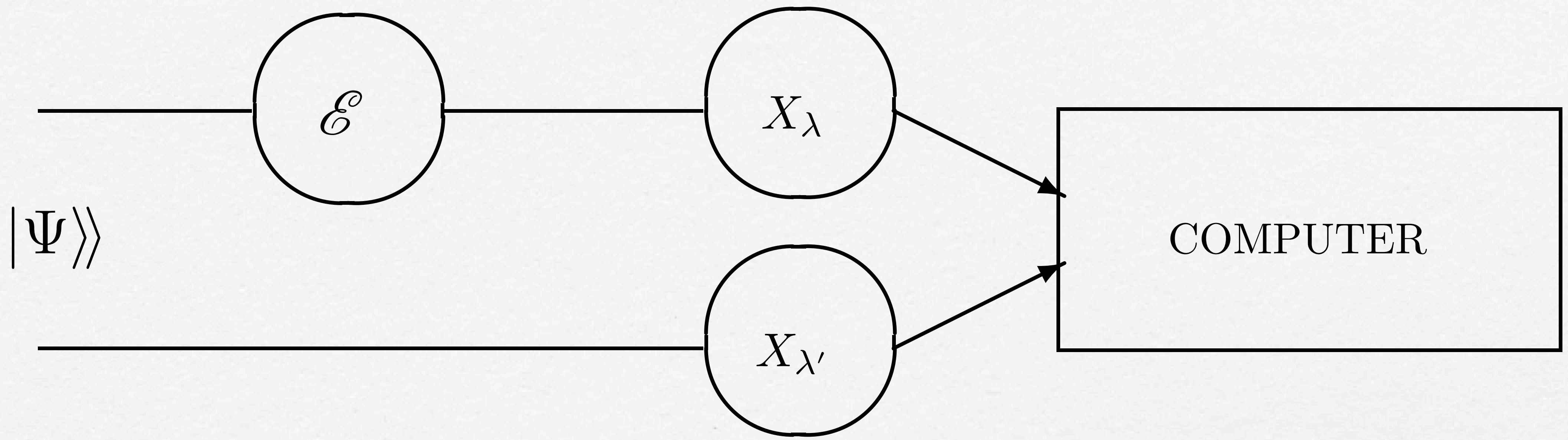
Tomography of QO's



- **General method:** Two identical quantum systems are prepared in an entangled state $|\Psi\rangle\rangle$. One of the two systems undergoes the quantum operation $\mathscr{E}$, whereas the other is left untouched. At the output one makes a quantum tomographic estimation, photocurrent by measuring jointly two random observables from a quorum $\{X_\lambda\}$.
- The output state is the joint density matrix

$$|\Psi\rangle\rangle\langle\langle\Psi| \to R(\Psi) \equiv \mathscr{E} \otimes \mathscr{I}(|\Psi\rangle\rangle\langle\langle\Psi|).$$

Tomography of QO's



- The quantum operation $\mathscr{E}$ is in correspondence with $R_{\mathscr{E}} \equiv R(\Psi)$ for $\Psi = I$, and for invertible Ψ the two matrices $R(I)$ and $R(\Psi)$ are connected as follows

$$R(I) = (I \otimes \Psi^{-1\intercal})R(\Psi)(I \otimes \Psi^{-1*}) \ .$$

Hence, the quantum operation (four-index) matrix $R_{\mathscr{E}}$ can be obtained by estimating via quantum tomography the following output ensemble averages

$$\langle\!\langle i,j|R(I)|l,k\rangle\!\rangle = \left\langle \ |l\rangle\langle i| \otimes \Psi^{-1*}|k\rangle\langle j|\Psi^{-1\intercal} \ \right\rangle \ .$$

Tomography of QO's

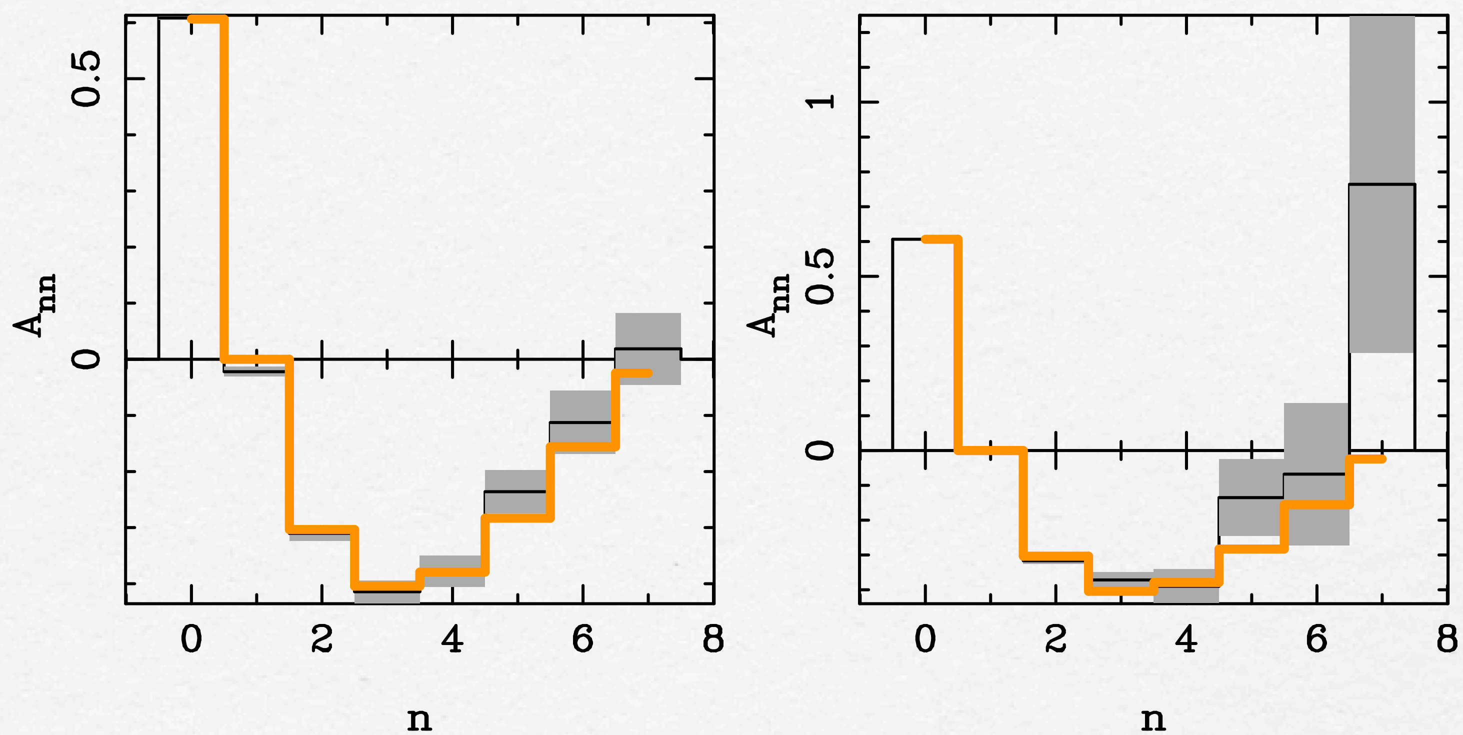


Figure 1: Homodyne tomography of the quantum operation A corresponding to the unitary displacement of one mode of the radiation field. Diagonal elements A_{nn} (shown by thin solid line on an extended abscissa range,) with their respective error bars in gray shade, compared to the theoretical probability (thick solid line). Similar results are obtained for all upper and lower diagonals of the quantum operation matrix A. The reconstruction has been achieved using an entangled state $|\psi\rangle\rangle$ at the input corresponding to parametric downconversion of vacuum with mean thermal photon $\bar{n}$ and quantum efficiency at homodyne detectors η. Top: $z = 1$, $\bar{n} = 5$, $\eta = 0.9$, and 150 blocks of 10^4 data have been used. Bottom: $z = 1$, $\bar{n} = 3$, $\eta = 0.7$, and 300 blocks of $2 \cdot 10^5$ data have been used. The bottom plot corresponds to the same parameters of the experiment in Ref. M. Vasilyev, S.-K. Choi, P. Kumar, and G. M. D'Ariano, Phys. Rev. Lett. **84** 2354 (2000).

Tomography of QO's

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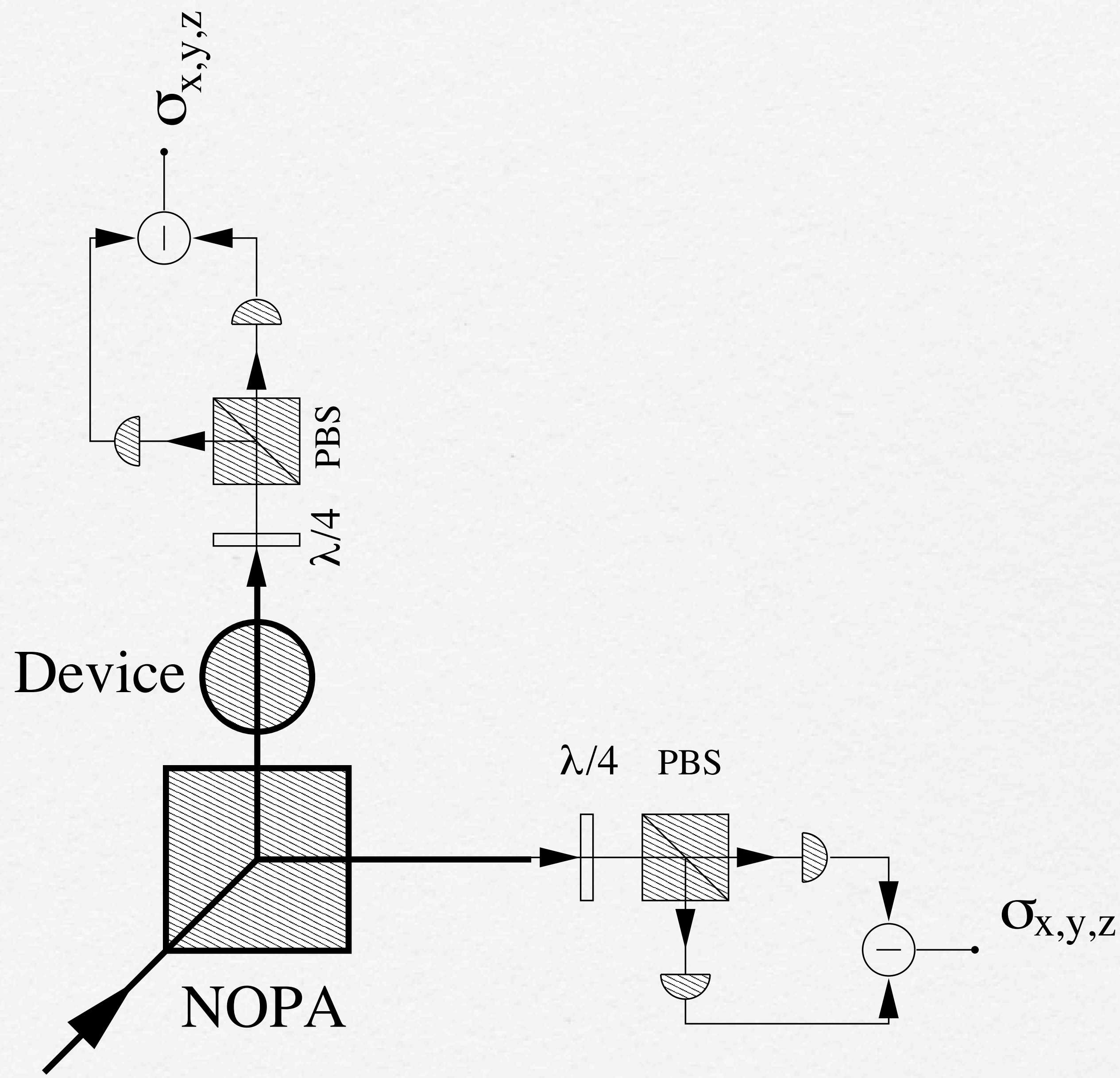
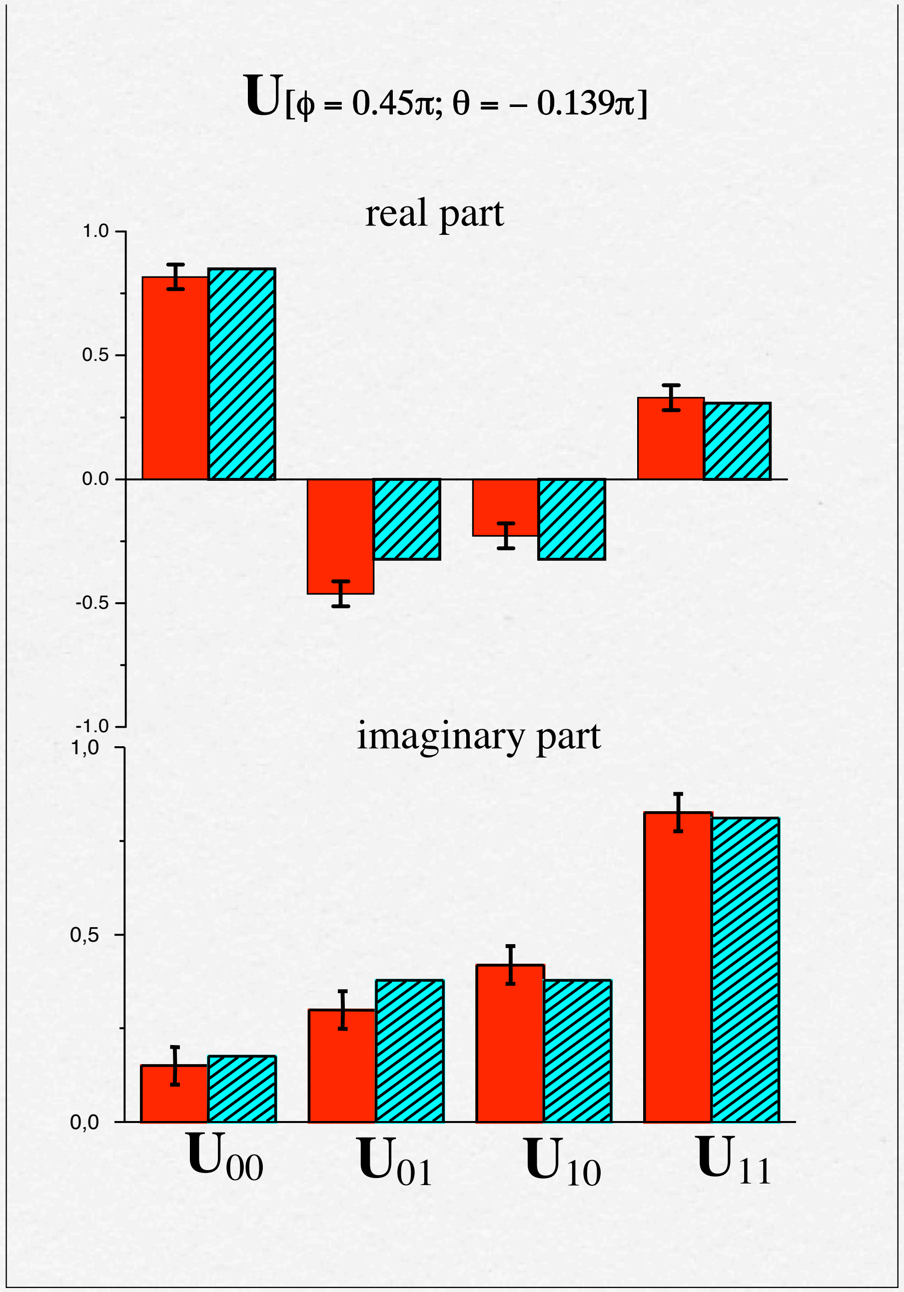
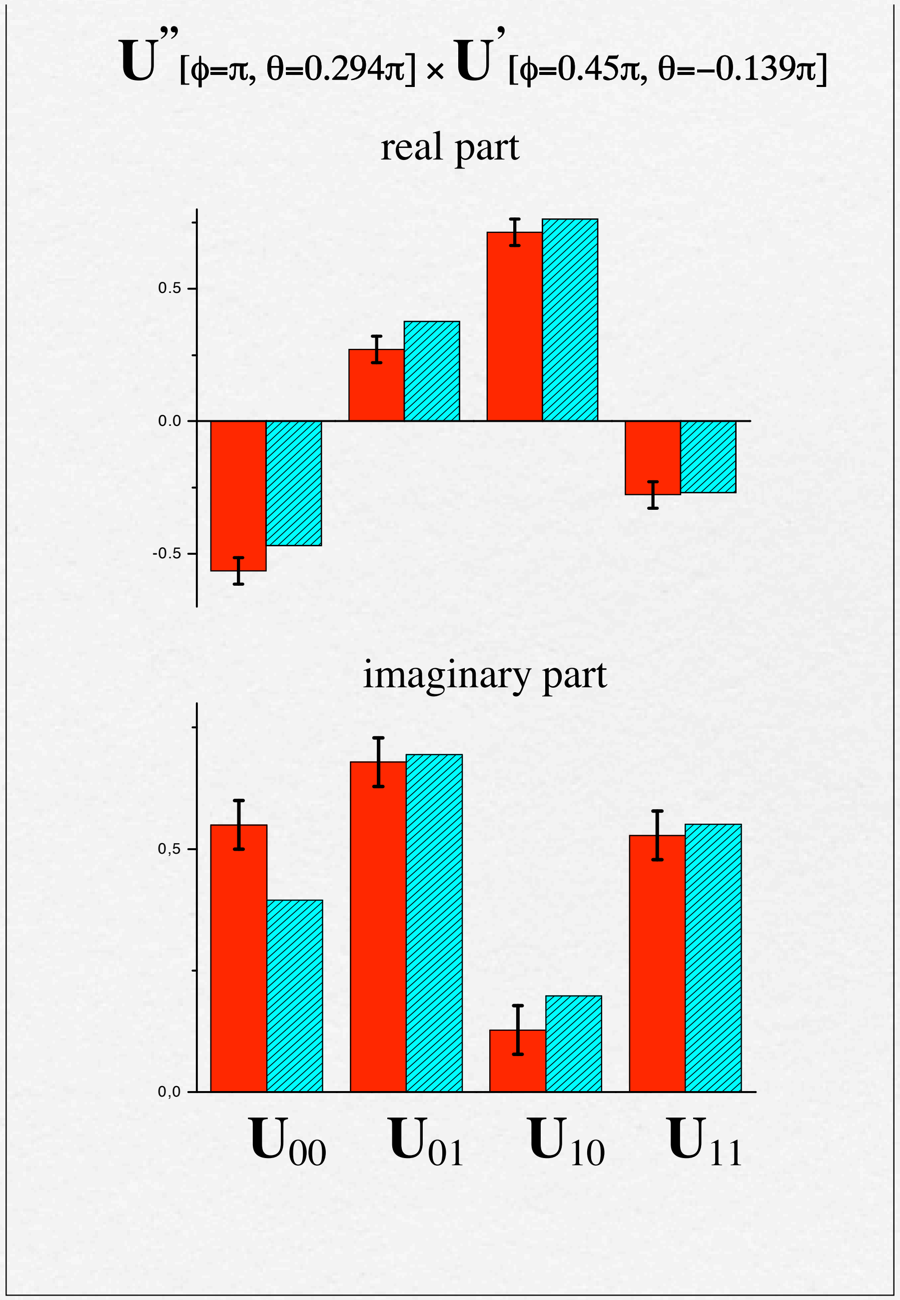


Figure 1: Experiment in progress in Roma La Sapienza, F. De Martini lab.

Tomography of QO's



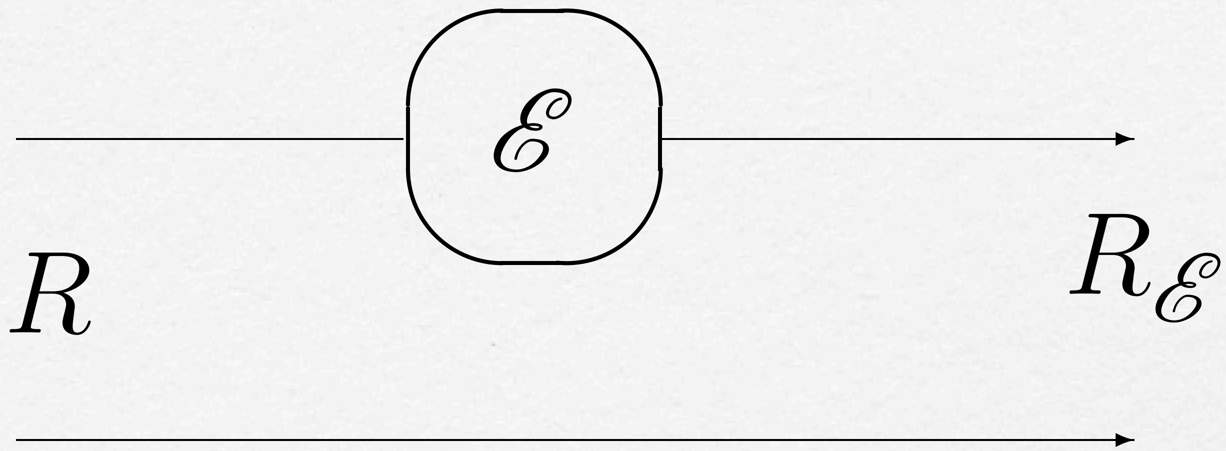


Faithful states

- Is it possible to characterize a quantum operation using mixed entangled states, or even separable ones?
- Answer: yes, as long as the state is *faithful*.

Faithful states

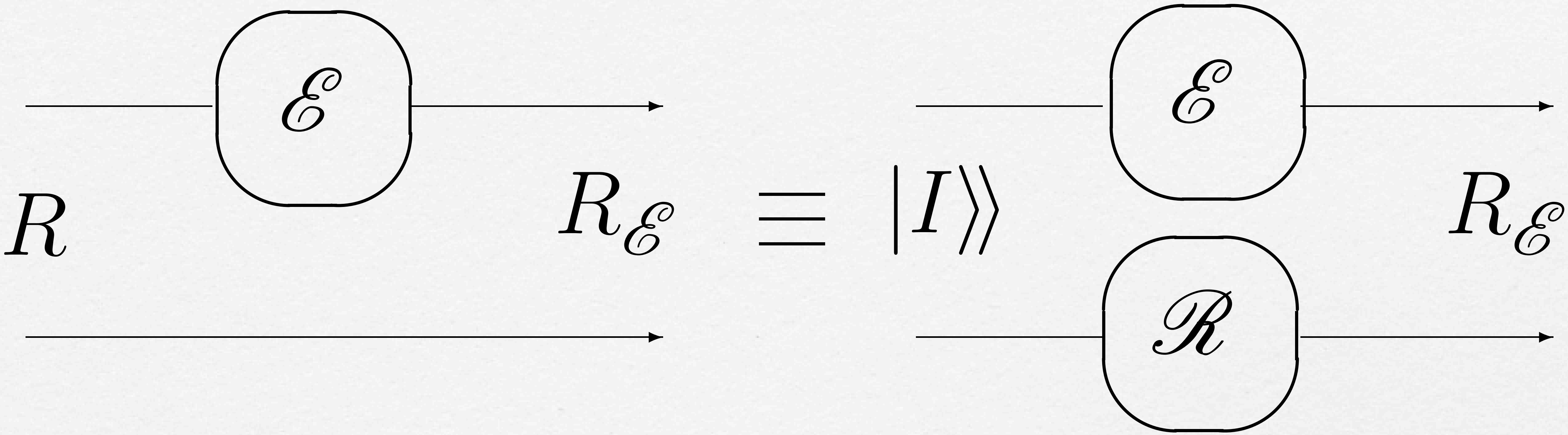
- We call a bipartite state faithful when acting with a channel on one of the two quantum systems, the output state carries a complete information about the channel.



$$R_{\mathscr{E}} \doteq \mathscr{E} \otimes \mathscr{I}(R).$$

Namely: the input state R is called *faithful* when the correspondence between the output state $R_{\mathscr{E}} \doteq \mathscr{E} \otimes \mathscr{I}(R)$ and the quantum channel $\mathscr{E}$ is one-to-one.

Faithful states



$$R = \sum_l |A_l\rangle\rangle\langle\langle A_l| = \mathscr{I} \otimes \mathscr{R}(|I\rangle\rangle\langle\langle I|), \ \mathscr{R}(\rho) = \sum_l A_l^{\intercal} \rho A_l^{*}.$$

- A state R is faithful when the map $\mathscr{R}$ is invertible, in order to guarantee the one-to-one correspondence between $R_{\mathscr{E}}$ and $\mathscr{E}$.

Faithful states

- The information about the channel $\mathscr{E}$ can be extracted from $R_{\mathscr{E}}$ as follows

$$\mathscr{E}(\rho) = \mathrm{Tr}_2[(I \otimes \rho^{\intercal})\mathscr{I} \otimes \mathscr{R}^{-1}(R_{\mathscr{E}})].$$

- A pure state $R \equiv |A\rangle\!\rangle\langle\!\langle A|$ is faithful iff it has maximal Schmidt's number.

- The set of faithful states R is *dense* within the set of all bipartite states.

- However, the knowledge of the map $\mathscr{E}$ from a measured $R_{\mathscr{E}}$ will be affected by increasingly large statistical errors for $\mathscr{R}$ approaching a non-invertible map.

- It follows that there are faithful states among mixed separable states.

Faithful states: continuous variables

- The inverse map $\mathscr{R}^{-1}$ is unbounded.

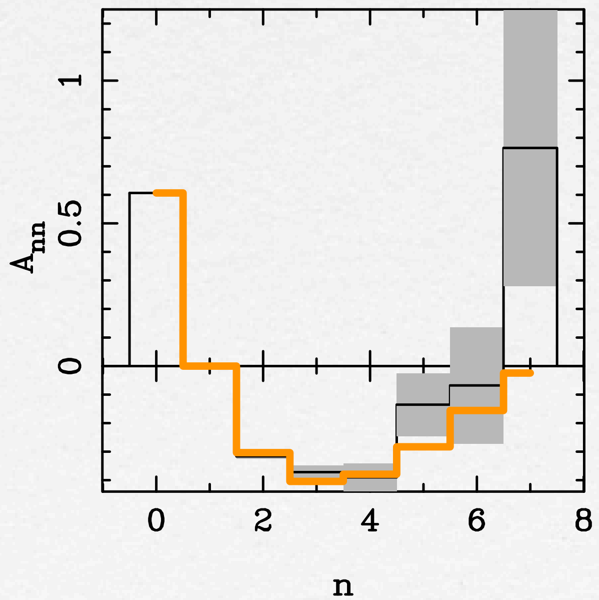
- As a result we will recover the channel $\mathscr{E}$ from the measured $R_{\mathscr{E}}$ with unbounded amplification of statistical errors, (depending on the chosen representation).

- Example: twin beam from parametric down-conversion of vacuum

$$|\Psi\rangle\rangle = \Psi \otimes I|I\rangle\rangle, \quad \Psi = (1-|\xi|^2)^{\frac{1}{2}}\xi^{a^\dagger a}, \qquad |\xi| < 1.$$

- The state is faithful, but the operator Ψ^{-1} is unbounded, whence the inverse map $\mathscr{R}^{-1}$ is also unbounded.

- For example, in a photon number representation $\mathsf{B} = \{|n\rangle\langle m|\}$, the effect will be an amplification of errors for increasing numbers n, m of photons.



Faithful states: continuous variables

- Consider now the quantum channel describing the *Gaussian displacement noise*

$$\mathscr{N}_\nu(\rho) = \int_{\mathbb{C}} \frac{\mathrm{d}\,\alpha}{\pi\nu} e^{-\frac{|\alpha|^2}{\nu}} D(\alpha)\rho D^\dagger(\alpha),$$

- analogous of the depolarizing channel for infinite dimension.

- Multiplication rule

$$\mathscr{N}_\nu \mathscr{N}_\mu = \mathscr{N}_{\nu+\mu},$$

  whence the inverse map is formally given by

$$\mathscr{N}_\nu^{-1} \equiv \mathscr{N}_{-\nu}.$$

- As a faithful state consider now the mixed state given by the twin-beam, with one beam spoiled by the Gaussian noise, namely

$$R = \mathscr{I} \otimes \mathscr{N}_\nu(|\Psi\rangle\!\rangle\langle\!\langle\Psi|).$$

Faithful states: continuous variables

- Unboundedness of the inverse map can wash out completely the information on the channel in some particular chosen representation.

- Example: (overcomplete) representation $\mathsf{B} = \{|\alpha\rangle\langle\beta|\}$, with $|\alpha\rangle$ and $|\beta\rangle$ coherent states.

- From the identity

$$\mathscr{N}_\nu(|\alpha\rangle\langle\alpha|) = \frac{1}{\nu+1} D(\alpha) \left(\frac{\nu}{\nu+1}\right)^{a^\dagger a} D^\dagger(\alpha),$$

one obtains

$$\mathscr{N}_\nu^{-1}(|\alpha\rangle\langle\alpha|) = \frac{1}{1-\nu} D(\alpha) \left(1-\nu^{-1}\right)^{-a^\dagger a} D^\dagger(\alpha),$$

- which has convergence radius $\nu \leq \frac{1}{2}$, which is the bound for Gaussian noise for the quantum tomographic reconstruction for coherent-state and Fock representations.

- Therefore, we say that the state is *formally* faithful, however, we are constrained to representations which are analytical for the inverse map $\mathscr{R}^{-1}$.

How we describe a measuring apparatus

A measuring apparatus with possible "outcomes" $\{n = 1, 2, \ldots\}$ is described by a set of operators (called POVM)

$$\mathbf{P} = \{P_n\},$$

which provide the probability $p(n)$ of each n for all possible states ρ via

$$p(n) = \mathrm{Tr}[P_n \rho] \quad \textbf{Born rule}$$

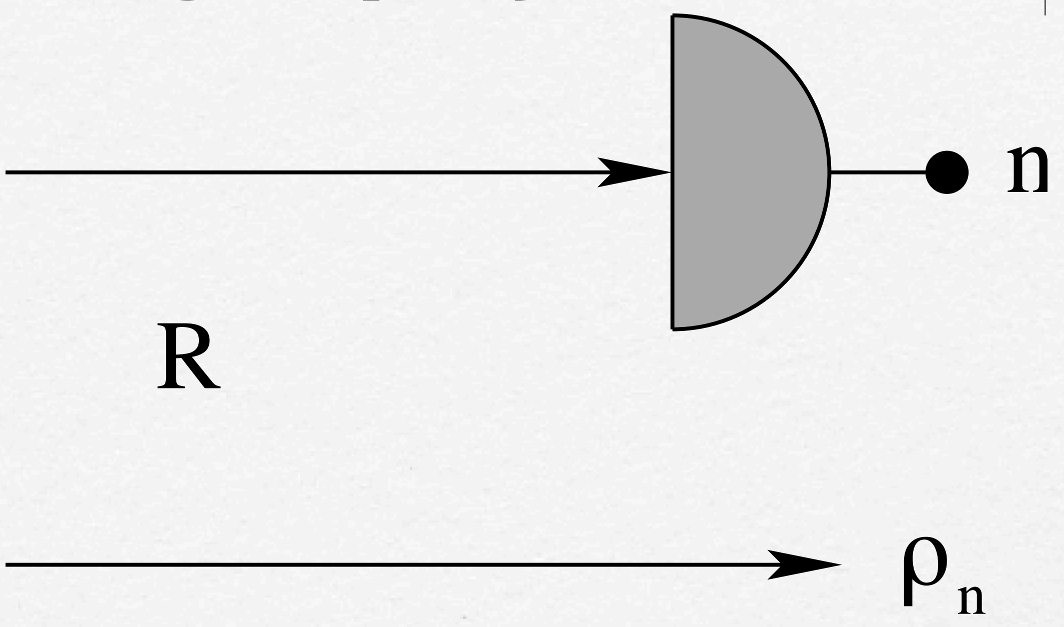
In order to have $p(n)$ a probability the operators P_n must satisfy the constraints

$$P_n \geq 0, \qquad \sum_n P_n = I.$$

Quantum calibration

In principle we can calibrate a quantum measuring apparatus without knowing its functioning, by determining experimentally its POVM

Tomography of POVM's



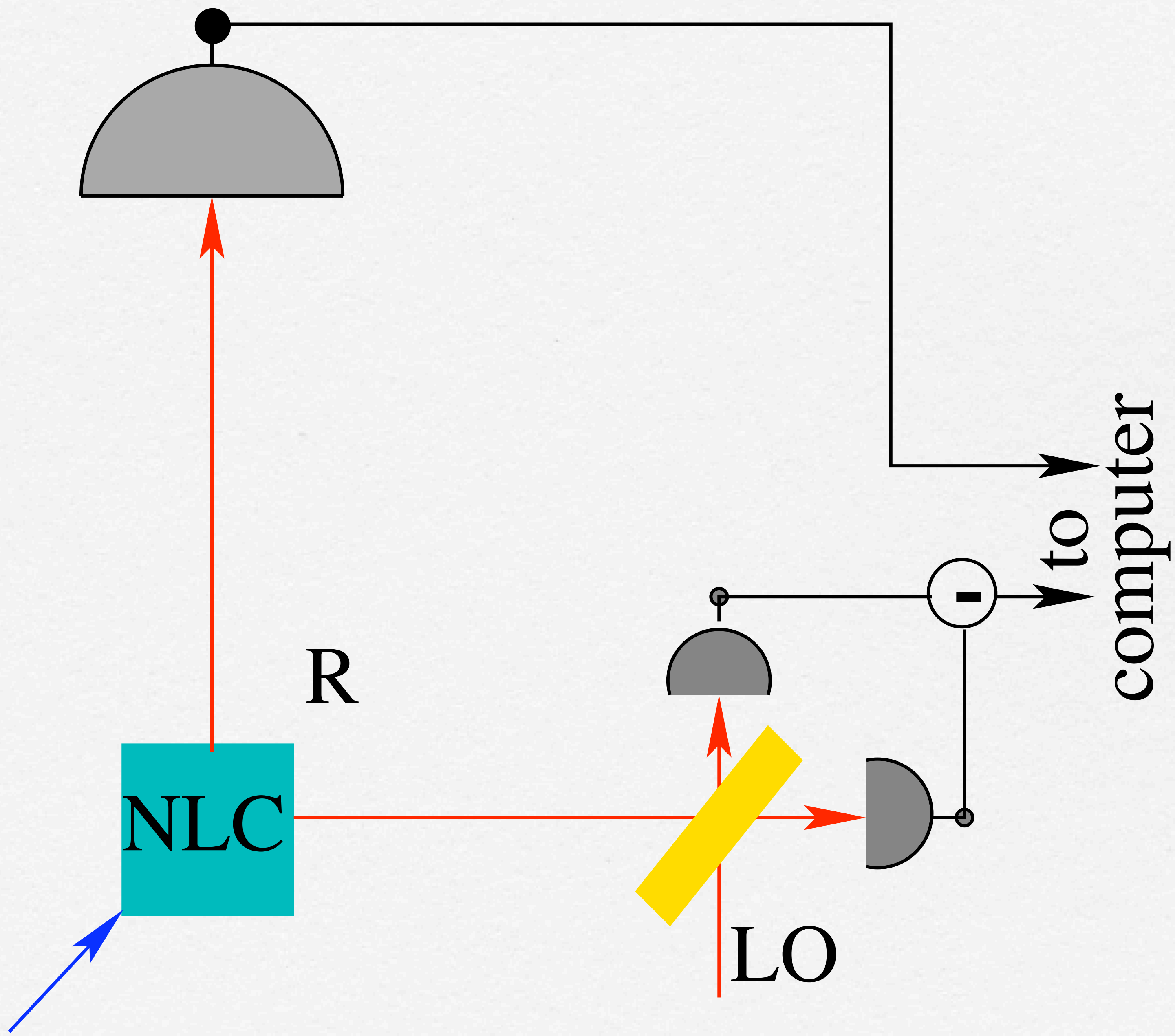
In terms of the POVM $\mathbf{P} \doteq \{P_n\}$ of the detector, the outcome n will occur with probability $p(n)$ corresponding to the conditioned state ρ_n given by

$$p(n) = \mathrm{Tr}[(P_n \otimes I)R], \qquad \rho_n = \frac{\mathrm{Tr}_1[(P_n \otimes I)R]}{\mathrm{Tr}[(P_n \otimes I)R]},$$

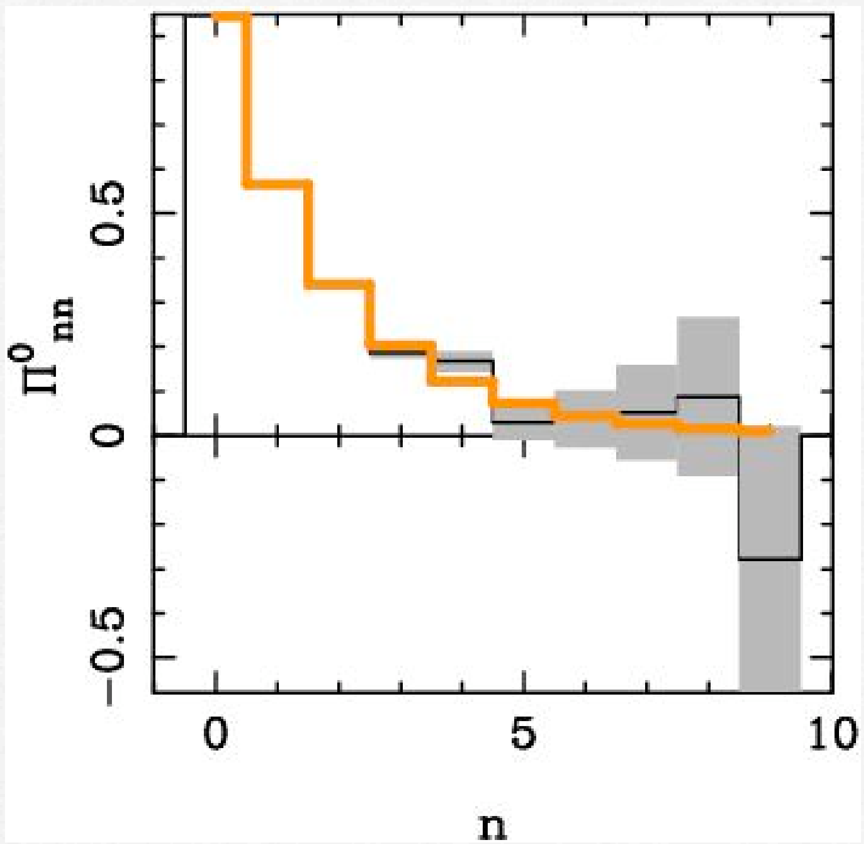
from which we can obtain the POVM as follows

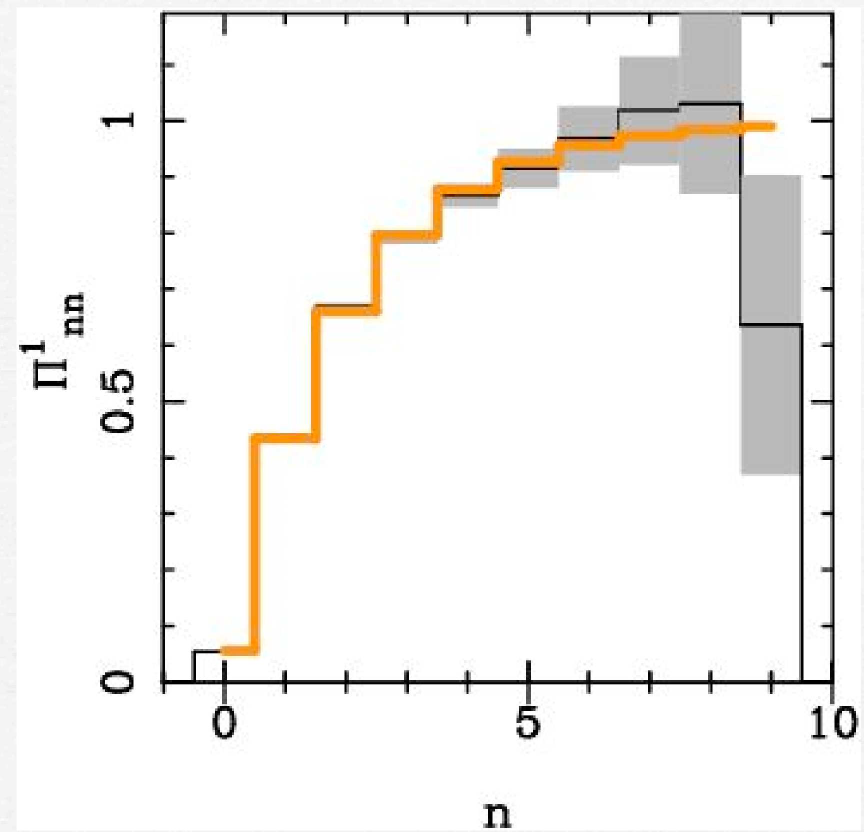
$$P_n = p(n)[\mathscr{R}^{-1}(\rho_n)]^{\mathsf{T}}, \quad \mathscr{R}(\rho) = \mathrm{Tr}_1[(\rho^{\mathsf{T}} \otimes I)R].$$

Quantum calibration of a photocounter



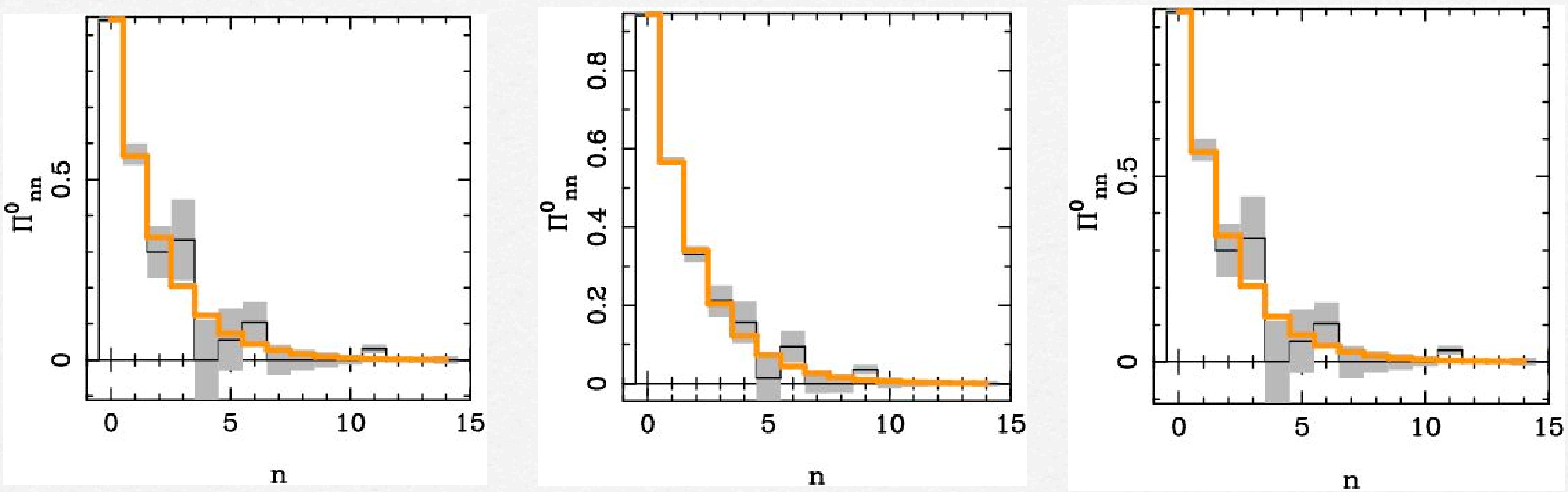
Tomography of POVM's





Homodyne tomography of an On/Off photo-detector with quantum efficiency $\eta = 0.4$ and thermal noise photon number $\nu = 0.1$. The reconstruction is obtained by pattern-function averaging of $1.5 \cdot 10^6$ data, for homodyne quantum efficiency $\eta = 0.9$ and twin beam thermal photon $\bar{n} = 3$.

Tomography of POVM's



Homodyne tomography of an On/Off photodetector with quantum efficiency $\eta = 0.4$ and thermal noise photon number $\nu = 0.1$, with $\bar{n} = 3$ photons in the twin-beam. The ML estimation of the diagonal of the only Off POVM element are reported for different values of sample size N and homodyne quantum efficiency η_H. Left: $N = 10^5$, $\eta_H = 0.7$; Middle: $N = 10^4$, $\eta_H = 0.9$; Right: $N = 10^6$, $\eta_H = 0.7$.

Summary of lecture I

We have seen:

- Short review of basic concepts (Hilbert Schmidt isomorphism, Quantum Operations, Complete Positivity, POVM's)
- Basic principles of Quantum Tomography
- Complete characterization of the Quantum Operation of a device using an entangled input and faithful states
- Quantum calibration of a measuring apparatus

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